Satellite and Optical Communication

BEC515D

MODULE 1

Satellite Orbits and Trajectories

What is a Satellite?

A satellite in general is any natural or artificial body moving around a celestial body such as planets and stars.

In the present context, reference is made only to artificial satellites orbiting the planet Earth.

These satellites are put into the desired orbit and have payloads depending upon the intended application.

ORBIT

- The path followed by the motion of a satellite around Earth is called an orbit.
- This is the path that is followed repeatedly periodically,

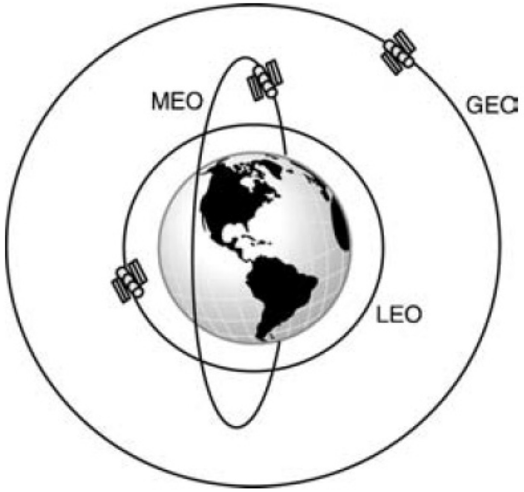


Figure 2.1 Example of orbital motion – satellites revolving around Earth

TRAJECTORY

- The path followed by a launch vehicle while launching the satellite is called the trajectory.
- The path followed by a rocket on its way to the right position for a satellite launch or the path followed by orbiting satellites when they move from an intermediate orbit to their final destined orbit are examples of trajectories.
- This is a path that is not periodically revisited.

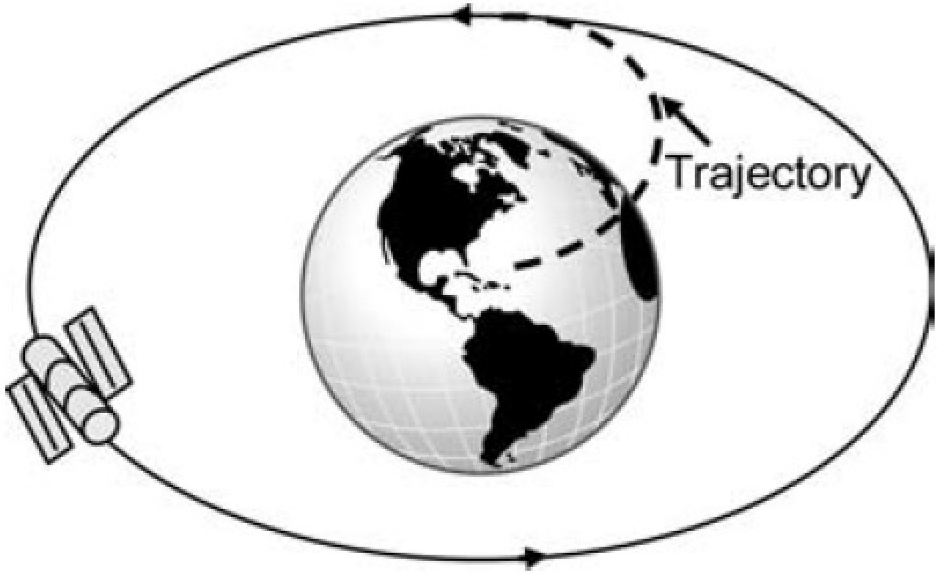


Figure 2.2 Example of trajectory – path followed by a rocket on its way during satellite launch

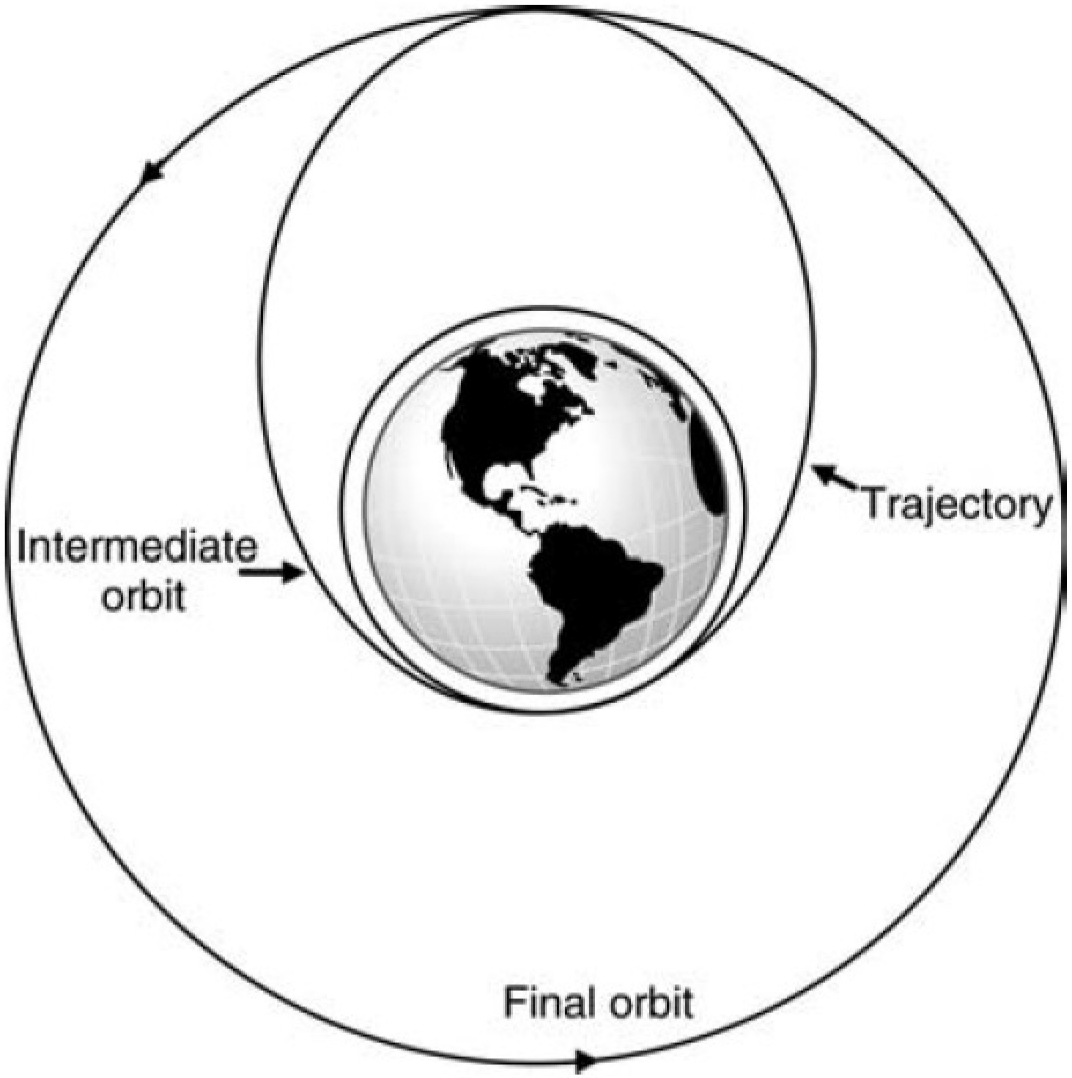
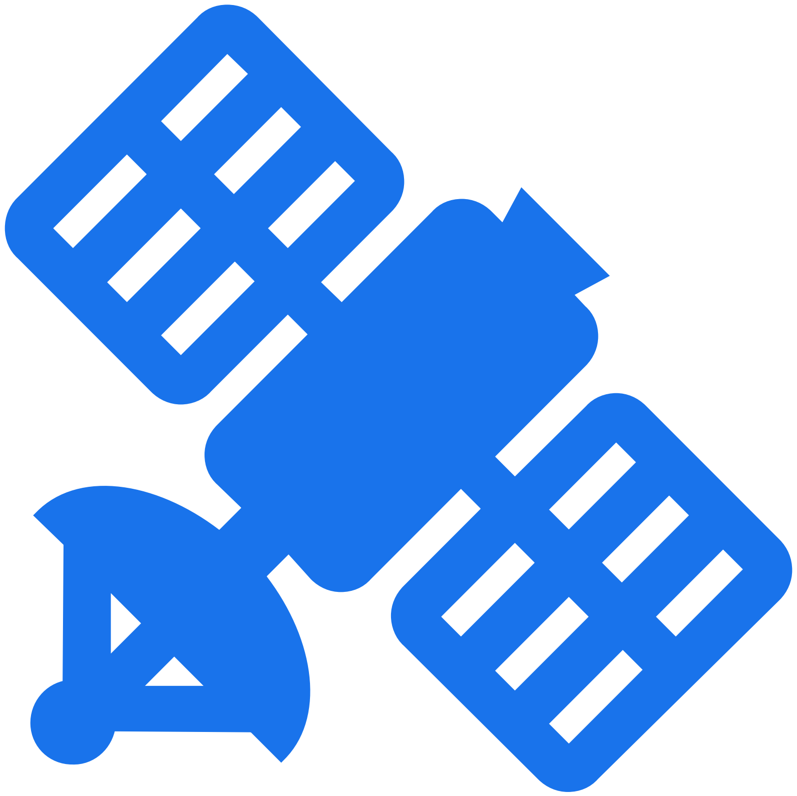


Figure 2.3 Example of trajectory – motion of a satellite from the intermediate orbit to the final orbit

Orbiting Satellites – Basic Principles

- The motion of natural and artificial satellites around Earth is governed by two forces.
  - **Centripetal force** directed towards the centre of the Earth due to the **gravitational force** of attraction of Earth
  - **Centrifugal force** that acts outwards from the centre of the Earth
    - Centrifugal force is the force exerted during circular motion, by the moving object upon the other object around which it is moving.



Newton's Law of Gravitation

- According to Newton's law of gravitation, every particle irrespective of its mass attracts every other particle with a gravitational force whose magnitude is directly proportional to the product of the masses of the two particles and inversely proportional to the square of the distance between them and written as

$$F = \frac{Gm_1m_2}{r^2}$$

where

$m_1, m_2 =$ masses of the two particles
$r =$ distance between the two particles
$G =$ gravitational constant $= 6.67 \times 10^{-11}\ \mathrm{m^3/kg\ s^2}$

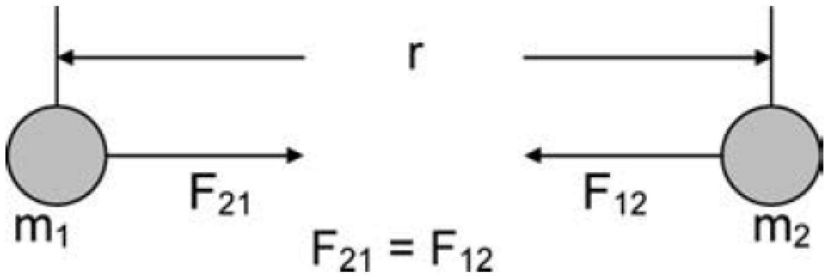


Figure 2.5 Newton's law of gravitation

Newton's Second Law of Motion

- According to Newton's second law of motion, the force equals the product of mass and acceleration.
- In the case of a satellite orbiting Earth, if the orbiting velocity is v, then the acceleration, called centripetal acceleration, experienced by the satellite at a distance r from the centre of the Earth would be v^2/r. If the mass of satellite is m, it would experience a reaction force of mv^2/r.
- This is the centrifugal force directed outwards from the centre of the Earth and for a satellite is equal in magnitude to the gravitational force.

Newton's Second Law of Motion

- If the satellite orbited Earth with a uniform velocity v, which would be the case when the satellite orbit is a circular one, then equating the two forces mentioned above would lead to an expression for the orbital velocity v as follows:

$$\frac{Gm_1m_2}{r^2} = \frac{m_2v^2}{r}$$

$$v = \sqrt{\left(\frac{Gm_1}{r}\right)} = \sqrt{\left(\frac{\mu}{r}\right)}$$

where

m_1 = mass of Earth
m_2 = mass of the satellite
$\mu = Gm_1 = 3.986013 \times 10^5\ \mathrm{km^3/s^2} = 3.986013 \times 10^{14}\ \mathrm{N\,m^2/kg}$

- The orbital period in such a case can be computed from

$$T = \frac{2\pi r^{3/2}}{\sqrt{\mu}}$$

Newton's Second Law of Motion

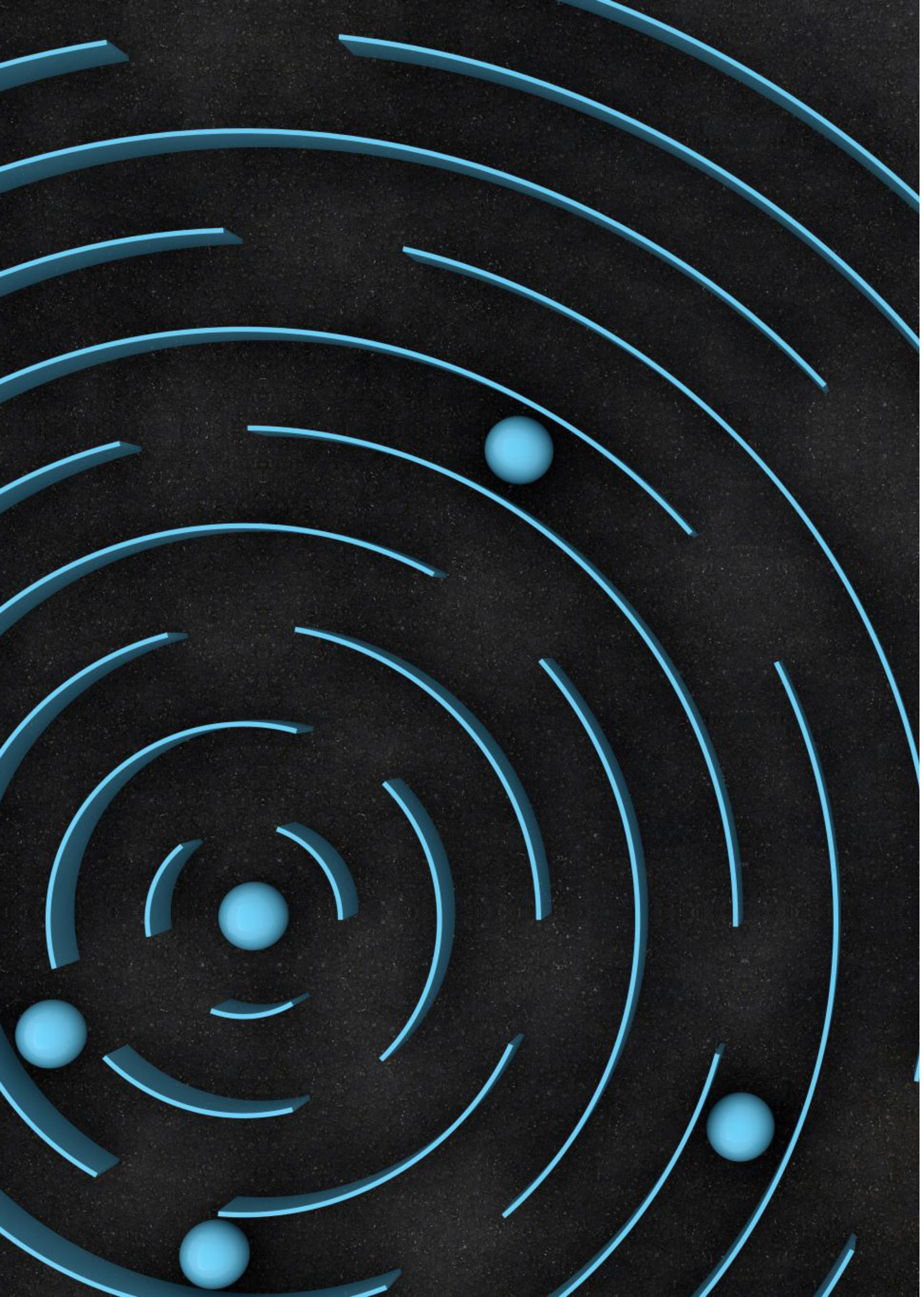
- In the case of an elliptical orbit, the forces governing the motion of the satellite are the same.
- The velocity at any point on an elliptical orbit at a distance $\boldsymbol{d}$ from the centre of the Earth is given by the formula

$$v = \sqrt{\left[\mu\left(\frac{2}{d} - \frac{1}{a}\right)\right]}$$

where
a = semi-major axis of the elliptical orbit

- The orbital period in the case of an elliptical orbit is given by

$$T = \frac{2\pi a^{3/2}}{\sqrt{\mu}}$$



Kepler's Laws

- Johannes Kepler, based on his lifetime study, gave a set of three empirical expressions that explained planetary motion.
- The movement of a satellite in an orbit is governed by three Kepler's laws.
- Though given for planetary motion, these laws are equally valid for the motion of natural and artificial satellites around Earth or for any body revolving around another body.

Kepler's First Law

- The orbit of a satellite around Earth is elliptical with the centre of the Earth lying at one of the foci of the ellipse.

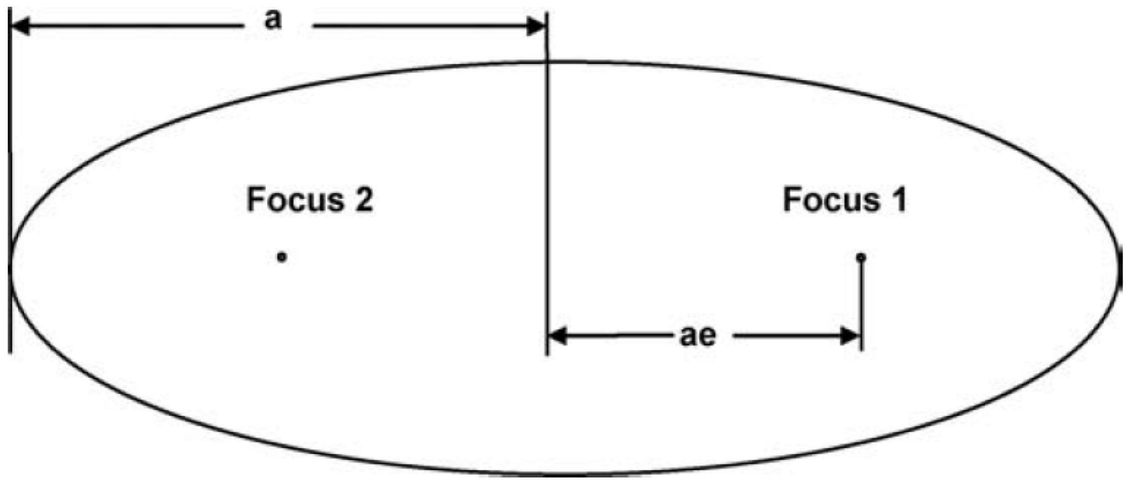


Figure 2.6 Kepler's first law

Kepler's First Law

- The elliptical orbit is characterized by its semi-major axis $\boldsymbol{a}$ and eccentricity $\boldsymbol{e}$.
- Eccentricity is the ratio of the distance between the centre of the ellipse and either of its foci $(= \ \boldsymbol{ae})$ to the semi-major axis of the ellipse $\boldsymbol{a}$.
- A circular orbit is a special case of an elliptical orbit where the foci merge together to give a single central point and the eccentricity becomes zero.

Kepler's First Law

- For any elliptical motion, the law of conservation of energy is valid at all points on the orbit.
- The law of conservation of energy states that energy can neither be created nor destroyed; it can only be transformed from one form to another.
- In the context of satellites, it means that the sum of the kinetic and the potential energy of a satellite always remains constant.
  - The value of this constant is equal to $-Gm_1m_2/(2a)$, where
    - m_1 = mass of Earth
    - m_2 = mass of the satellite
    - a = semi-major axis of the orbit

Kepler's First Law

- The kinetic and potential energies of a satellite at any point at a distance *r* from the centre of the Earth are given by

$$\text{Kinetic energy} = \frac{1}{2}(m_2 v^2)$$

$$\text{Potential energy} = -\frac{Gm_1 m_2}{r}$$

Therefore,

$$\frac{1}{2}(m_2 v^2) - \frac{Gm_1 m_2}{r} = -\frac{Gm_1 m_2}{2a}$$

$$v^2 = Gm_1 \left(\frac{2}{r} - \frac{1}{a}\right)$$

$$v = \sqrt{\left[\mu \left(\frac{2}{r} - \frac{1}{a}\right)\right]}$$

Kepler's Second Law

- The line joining the satellite and the centre of the Earth sweeps out equal areas in the plane of the orbit in equal time intervals.
  - i.e. the rate (dA/dt) at which it sweeps area A is constant.
- The rate of change of the swept-out area is given by

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\text{angular momentum of the satellite}}{2m}$$

where m is the mass of the satellite.

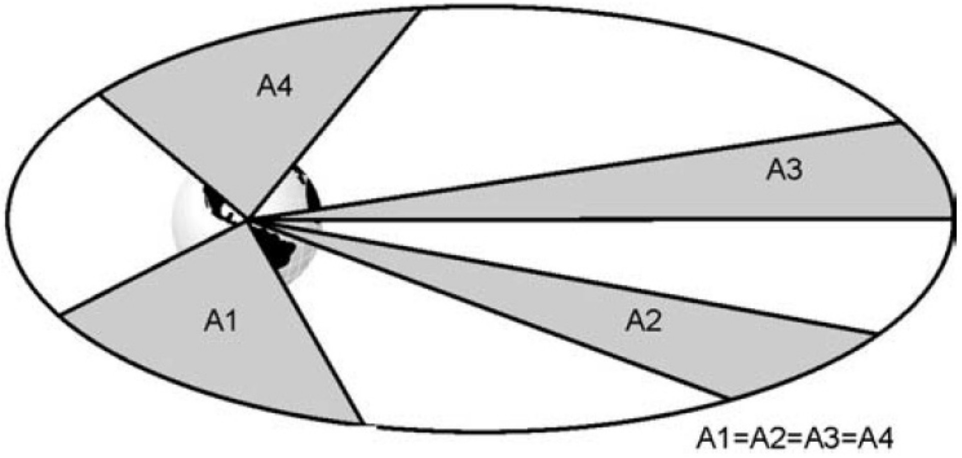


Figure 2.7 Kepler's second law

Kepler's Second Law

- Kepler's second law is also equivalent to the law of ***conservation of momentum***, which implies that the angular momentum of the orbiting satellite given by the product of the radius vector and the component of linear momentum perpendicular to the radius vector is constant at all points on the orbit.
- The angular momentum of the satellite of mass m is given by $mr^2\omega$, where ω is the angular velocity of the satellite.
- This further implies that the product $mr^2\omega = (m\omega r)\,(r) = mv'r$ remains constant.

Kepler's Second Law

- Here v' is the component of the satellite's velocity v in the direction perpendicular to the radius vector and is expressed as $v \cos \gamma$, where γ is the angle between the direction of motion of the satellite and the local horizontal, which is in the plane perpendicular to the radius vector r.
- This leads to the conclusion that the product $rv \cos \gamma$ is constant.

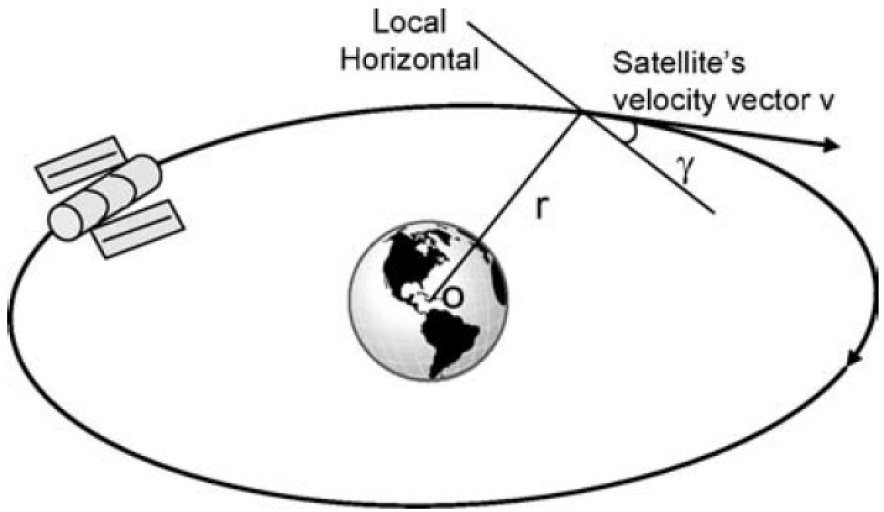


Figure 2.8 Satellite's position at any given time

Kepler's Second Law

- The product reduces to rv in the case of circular orbits and also at apogee and perigee points in the case of elliptical orbits due to angle γ becoming zero.
- The velocity component v is inversely proportional to the distance r.
- This implies that the satellite is at its lowest speed at the apogee point and the highest speed at the perigee point.
- In other words, for any satellite in an elliptical orbit, the dot product of its velocity vector and the radius vector at all points is constant.
- Hence,

$$v_p r_p = v_a r_a = vr \cos \gamma$$

Kepler's Second Law

$$v_p r_p = v_a r_a = vr \cos \gamma$$

where

v_p = velocity at the perigee point
r_p = perigee distance
v_a = velocity at the apogee point
r_a = apogee distance
v = satellite velocity at any point in the orbit
r = distance of the point
γ = angle between the direction of motion of the satellite and the local horizontal

Kepler's Third Law

- According to the Kepler's third law, also known as the *law of periods*, the square of the time period of any satellite is proportional to the cube of the semi-major axis of its elliptical orbit.
- Assuming a circular orbit with radius r, equating the gravitational force with the centrifugal force gives

$$\frac{Gm_1m_2}{r^2} = \frac{m_2v^2}{r}$$

Replacing v by ωr in the above equation gives

$$\frac{Gm_1m_2}{r^2} = \frac{m_2\omega^2r^2}{r} = m_2\omega^2r$$

$$\omega^2 = Gm_1/r^3$$

Kepler's Third Law

Substituting $\omega = 2\pi/T$ gives

$$T^2 = \left(\frac{4\pi^2}{Gm_1}\right) r^3$$

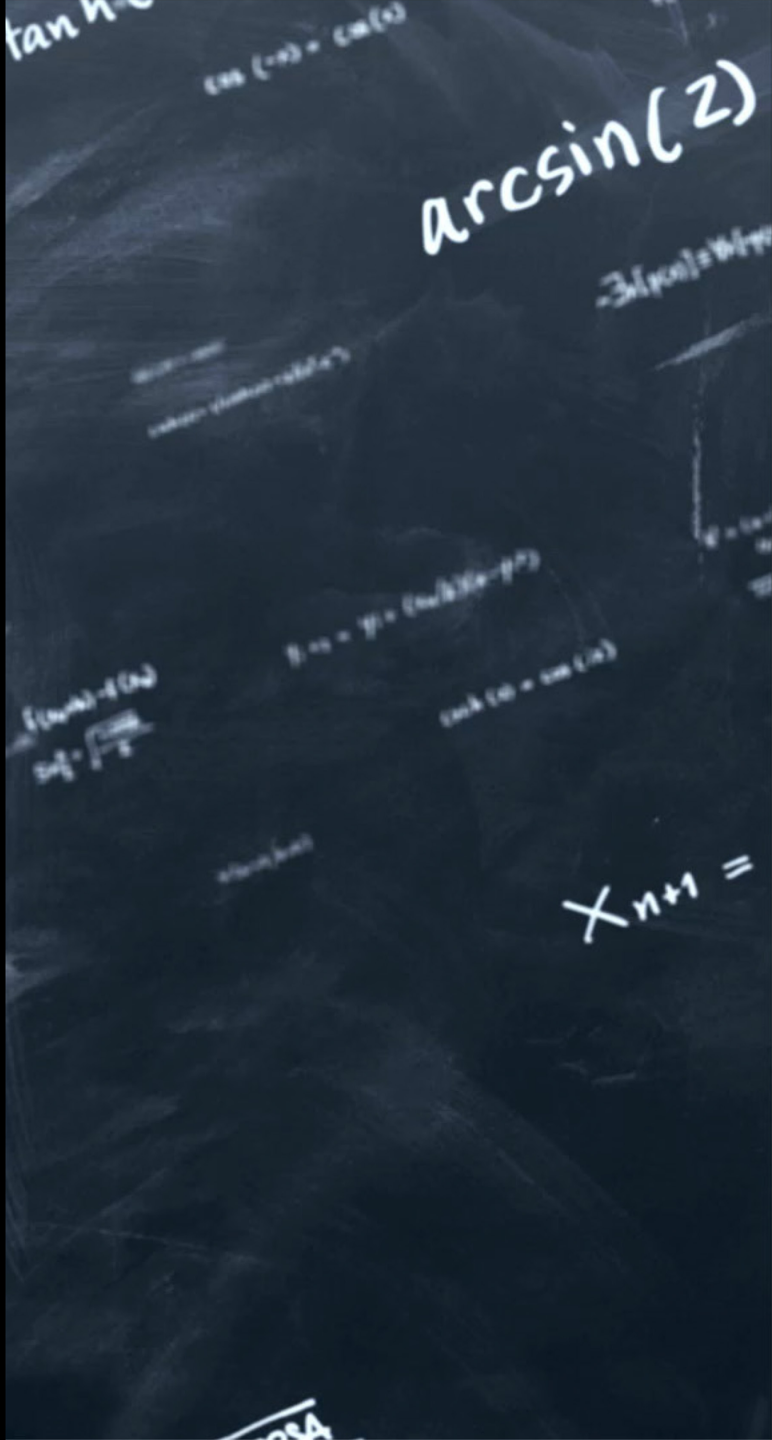
This can also be written as

$$T = \left(\frac{2\pi}{\sqrt{\mu}}\right) r^{3/2}$$

The above equation holds good for elliptical orbits provided r is replaced by the semi-major axis a. This gives the expression for the time period of an elliptical orbit as

$$T = \left(\frac{2\pi}{\sqrt{\mu}}\right) a^{3/2}$$

Orbital Parameters



Ascending and descending nodes

- The satellite orbit cuts the equatorial plane at two points:
- Descending node (N1), where the satellite passes from the northern hemisphere to the southern hemisphere,
- Ascending node (N2), where the satellite passes from the southern hemisphere to the northern hemisphere

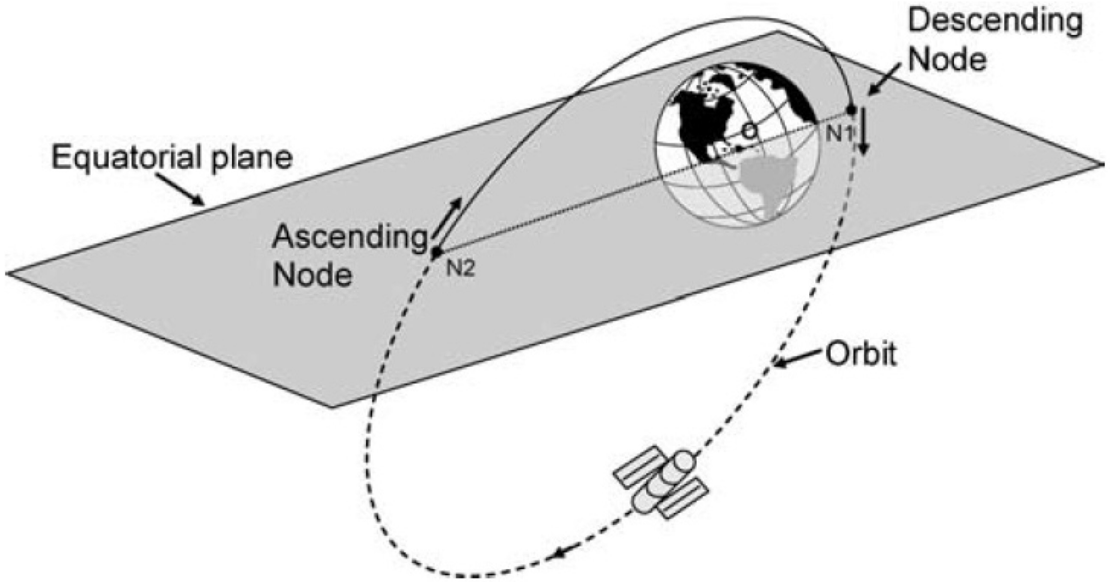


Figure 2.9 Ascending and descending nodes

Equinoxes

- The inclination of the equatorial plane of Earth with respect to the direction of the sun, defined by the angle formed by the line joining the centre of the Earth and the sun with the Earth's equatorial plane follows a sinusoidal variation and completes one cycle of sinusoidal variation over a period of 365 days.

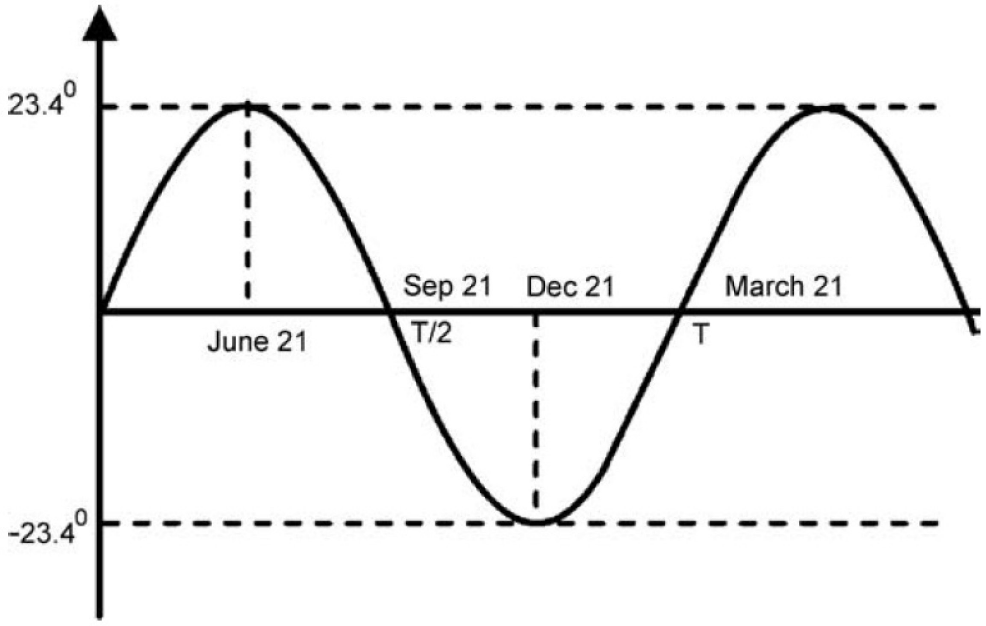


Figure 2.10 Yearly variation of angular inclination of Earth with the sun

Equinoxes

- The sinusoidal variation of the angle of inclination is defined by

$$\text{Inclination angle (in degrees)} = 23.4\ \sin\left(\frac{2\pi t}{T}\right)$$

  where *T* is 365 days.

- This expression indicates that the inclination angle is zero for $t = T/2$ and T .
- This is observed to occur on **20–21 March**, called the **spring equinox**, and **22–23 September**, called the **autumn equinox**.
- During the equinoxes, it can be seen that the equatorial plane of Earth will be aligned with the direction of the sun.

Equinoxes

- The line of intersection of the Earth's equatorial plane and the Earth's orbital plane that passes through the centre of the Earth is known as the *line of equinoxes*.
- The direction of this line with respect to the direction of the sun on 20-21 March determines a point at infinity called the vernal equinox (Y)

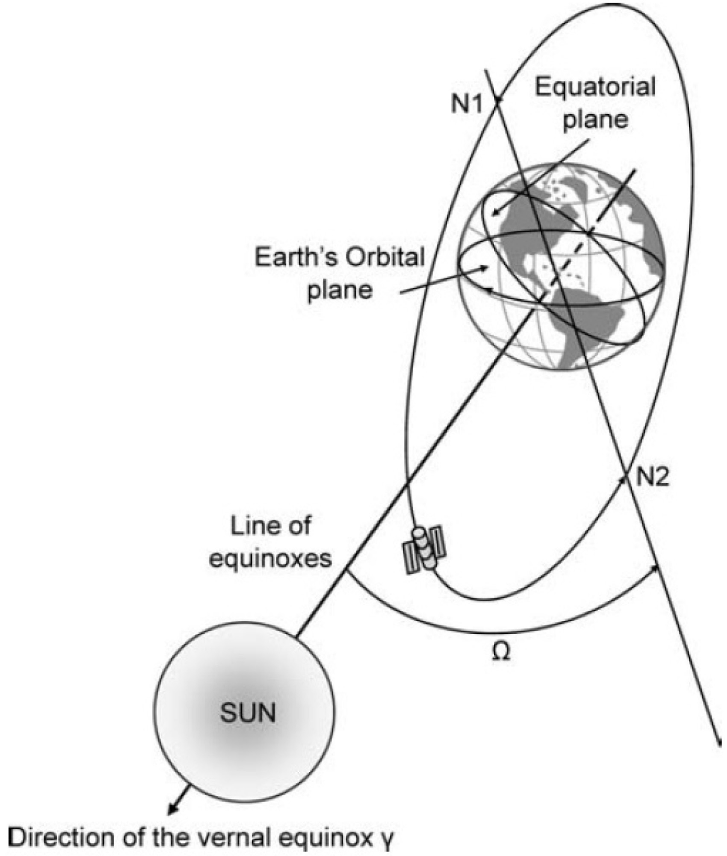


Figure 2.11 Vernal equinox

Solstices

- Solstices are the times when the inclination angle is at its maximum, i.e. 23.4°.
- These also occur twice during a year
  - on **20–21 June**, called the **summer solstice**,
  - on **21–22 December**, called the **winter solstice**.

Apogee

- Apogee is the point on the satellite orbit that is at the farthest distance from the centre of the Earth.

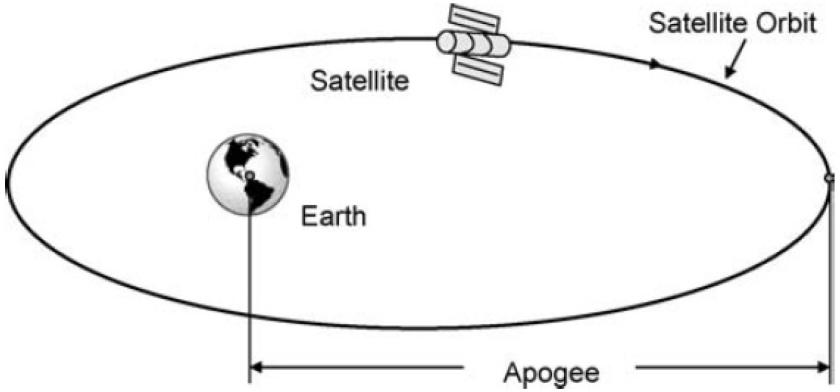


Figure 2.12 Apogee

- The apogee distance can be computed from the known values of the orbit eccentricity e and the semi-major axis a from

$$\text{Apogee distance} = a\,(1 + e)$$

Perigee

- Perigee is the point on the orbit that is nearest to the centre of the Earth.

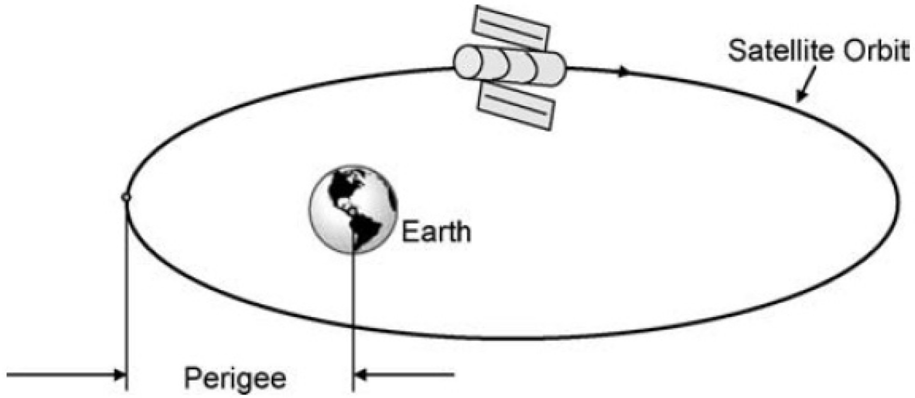


Figure 2.13 Perigee

- The perigee distance can be computed from the known values of orbit eccentricity e and the semi-major axis a from

$$\text{Perigee distance} = a\,(1 - e)$$

Eccentricity

- The orbit eccentricity $\boldsymbol{e}$ is the ratio of the distance between the centre of the ellipse and the centre of the Earth to the semi-major axis of the ellipse.

$$e = \frac{\text{apogee} - \text{perigee}}{\text{apogee} + \text{perigee}}$$

$$e = \frac{\text{apogee} - \text{perigee}}{2a}$$

Thus $e = \sqrt{(a^2 - b^2)}/a$, where a and b are semi-major and semi-minor axes respectively

Semi-major axis

- This is a geometrical parameter of an elliptical orbit.
- It can, however, be computed from known values of apogee and perigee distances as

$$a = \frac{\text{apogee} + \text{perigee}}{2}$$

Right ascension of the ascending node

- The right ascension of the ascending node is the line joining the ascending and descending nodes, with respect to the direction of the vernal equinox.
  - It tells about the orientation of the line of nodes.
- It is expressed as an angle measured from the vernal equinox towards the line of nodes in the direction of rotation of Earth.
- The angle could be anywhere from 0∘ to 360∘.

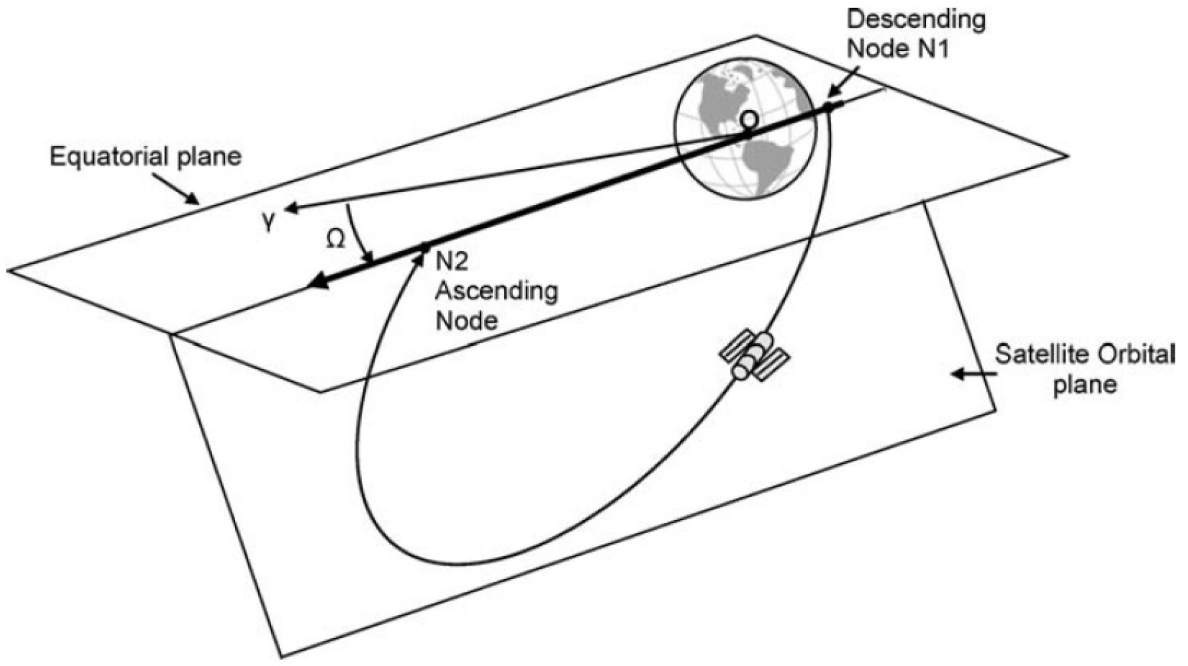


Figure 2.14 Right ascension of the ascending node

Right ascension of the ascending node

- Acquisition of the correct angle of right ascension of the ascending node (Ω) is important to ensure that the satellite orbits in the given plane.
- This can be achieved by choosing an appropriate injection time depending upon the longitude.
- Angle Ω can be computed as the difference between two angles:
  - Angle α between the direction of the vernal equinox and the longitude of the injection point
  - Angle β between the line of nodes and the longitude of the injection point
    - Angle β can be computed from

$$\sin\beta = \frac{\cos i \ \sin l}{\cos l \ \sin i}$$

where $\angle i$ is the orbit inclination and l is the latitude at the injection point.

Right ascension of the ascending node

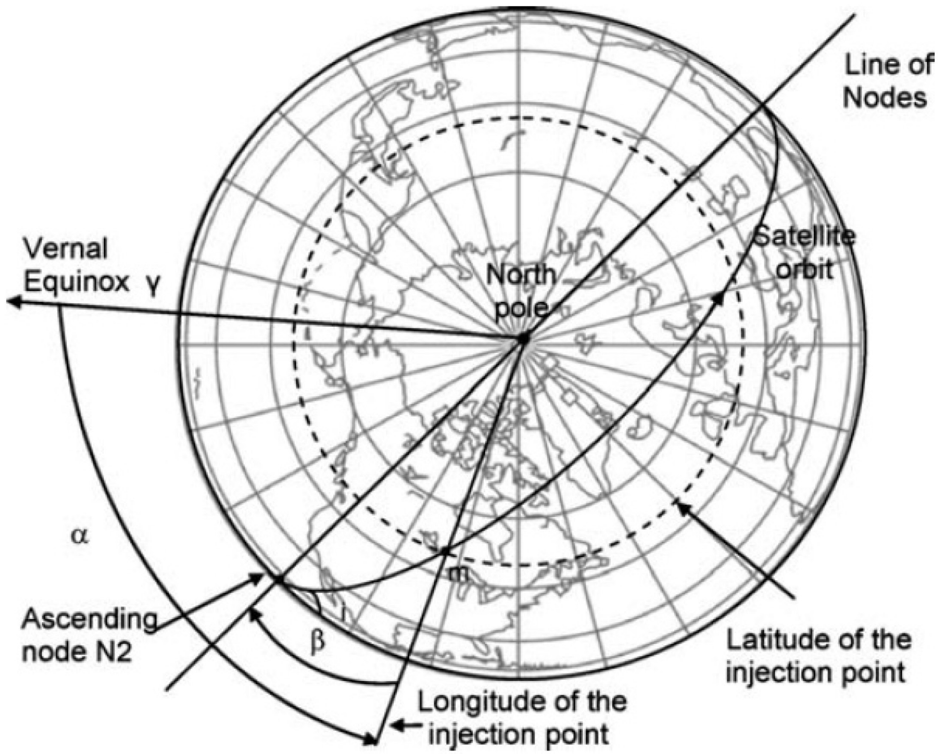


Figure 2.15 Computation of the right ascension of the ascending node

Inclination

- Inclination is the angle that the orbital plane of the satellite makes with the Earth's equatorial plane.
- The line of nodes divides both the Earth's equatorial plane as well as the satellite's orbital plane into two halves.
- Inclination is measured as the angle between that half of the satellite's orbital plane containing the trajectory of the satellite from the descending node to the ascending node to that half of the Earth's equatorial plane containing the trajectory of a point on the equator from n1 to n2, where n1 and n2 are respectively the points vertically below the descending and ascending nodes.

Inclination

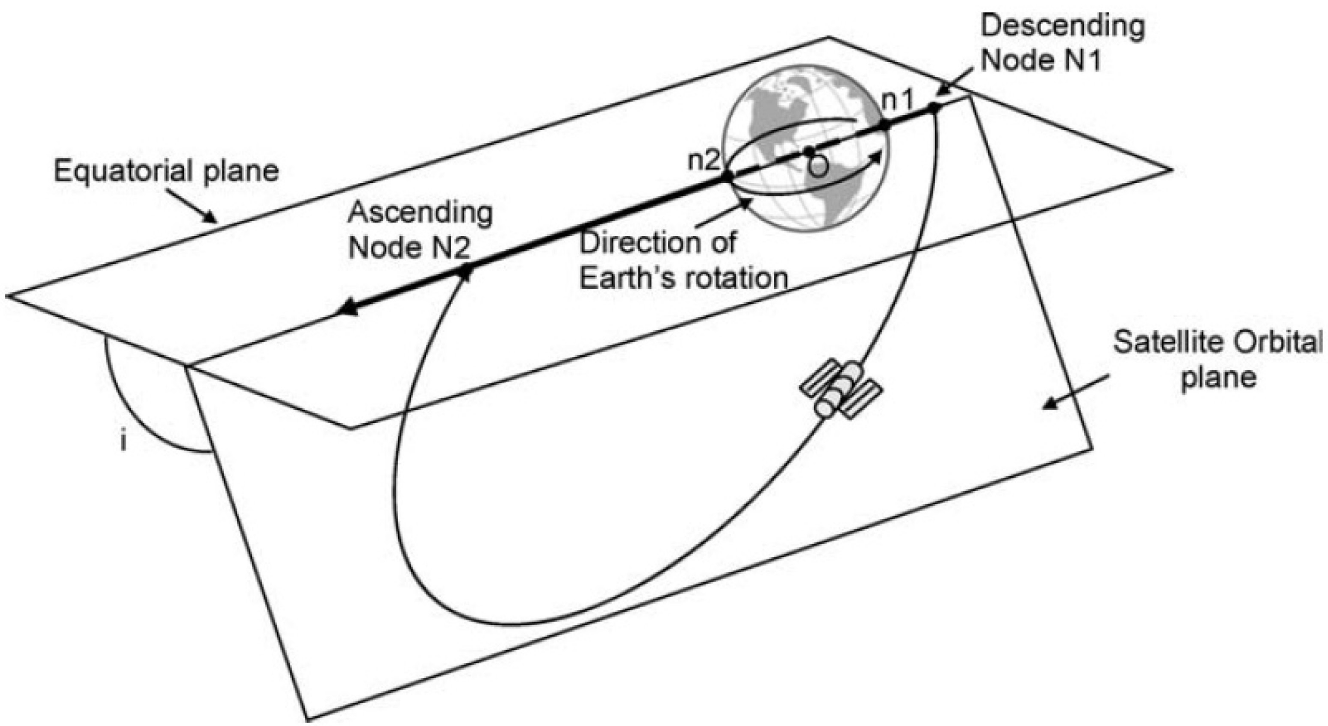


Figure 2.16 Angle of inclination

Inclination

- The inclination angle can be determined from the latitude l at the injection point and the angle A_z between the projection of the satellite's velocity vector on the local horizontal and north.
- It is given by

$$\cos i = \sin A_z \cos l$$

Argument of the perigee

- This parameter defines the location of the major axis of the satellite orbit.
- It is measured as the angle ω between the line joining the perigee and the centre of the Earth and the line of nodes from the ascending node to the descending node in the same direction as that of the satellite orbit.

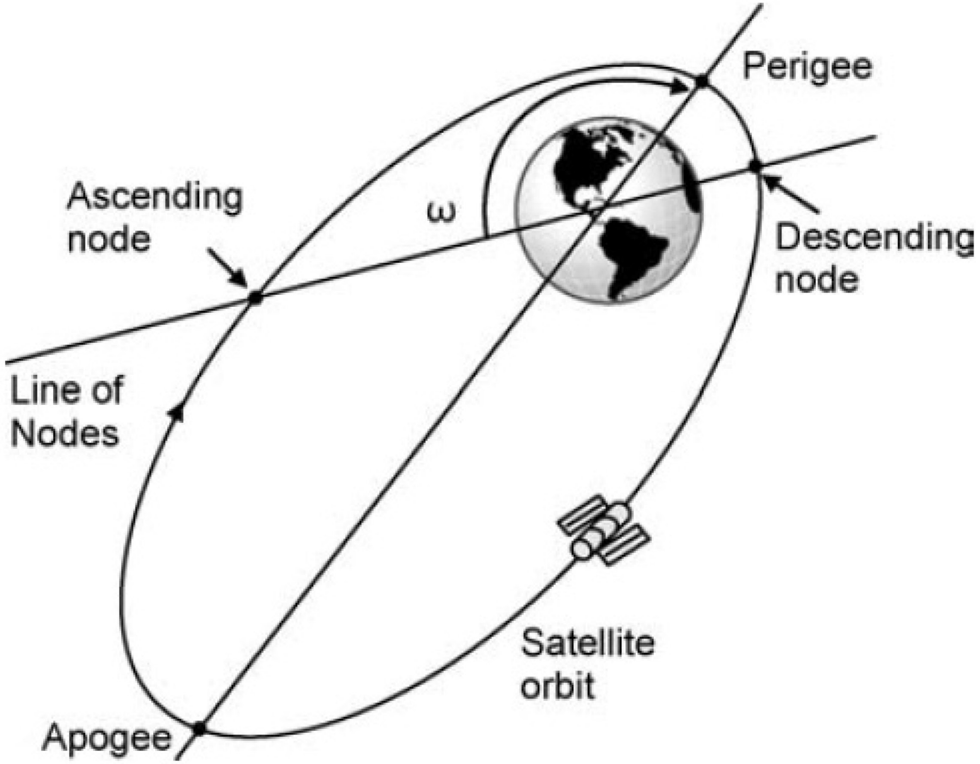


Figure 2.17 Argument of perigee

True anomaly of the satellite

- This parameter is used to indicate the position of the satellite in its orbit.
- This is done by defining an angle θ, called the true anomaly of the satellite, formed by the line joining the perigee and the centre of the Earth with the line joining the satellite and the centre of the Earth.

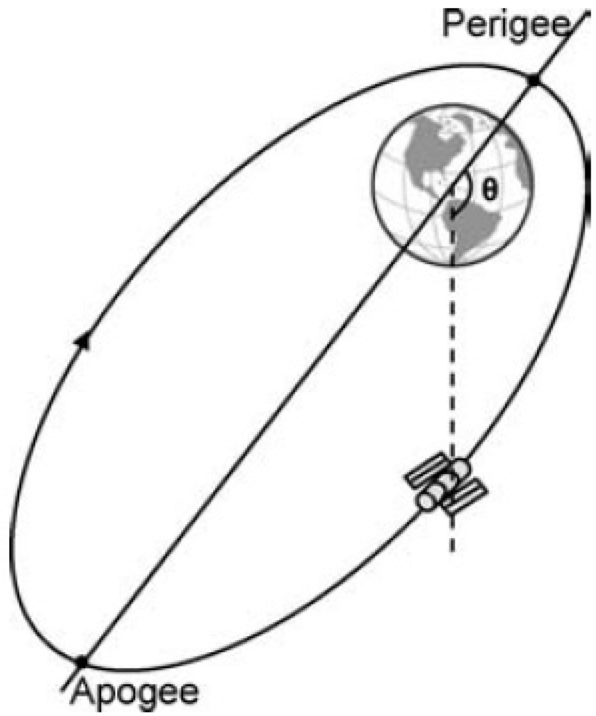


Figure 2.18 True anomaly of a satellite

Angles defining the direction of the satellite

- The direction of the satellite is defined by two angles:
  - Angle γ between the direction of the satellite's velocity vector and its projection in the local horizontal
  - Angle A_z between the north and the projection of the satellite's velocity vector on the local horizontal

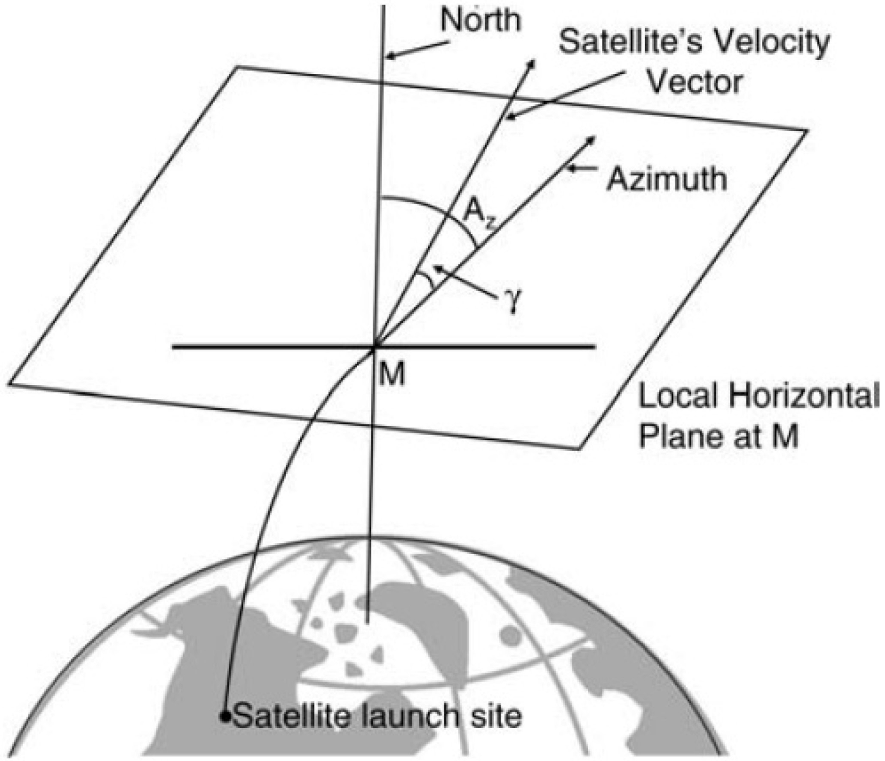


Figure 2.19 Angles defining the direction of the satellite

Problem 2.1

- A satellite is orbiting Earth in a uniform circular orbit at a height of $630\ km$ from the surface of Earth. Assuming the radius of Earth and its mass to be $6370\ km$ and $5.98 \times 10^{24}\ kg$ respectively, determine the velocity of the satellite (Take the gravitational constant $G = 6.67 \times 10^{-11} Nm^2/kg^2$).

Solution:

Orbit radius $R = 6370 + 630 = 7000\,\text{km} = 7\,000\,000\,\text{m}$

Also, constant $\mu = GM = 6.67 \times 10^{-11} \times 5.98 \times 10^{24}$

$$= 39.8 \times 10^{13}\ \text{N m}^2/\text{kg}$$
$$= 39.8 \times 10^{13}\ \text{m}^3/\text{s}^2$$

The velocity of the satellite can be computed from

$$V = \sqrt{\left(\frac{\mu}{R}\right)} = \sqrt{\left(\frac{39.8 \times 10^{13}}{7\,000\,000}\right)} = 7.54\,\text{km/s}$$

Problem 2.2

- The apogee and perigee distances of a satellite orbiting in an elliptical orbit are respectively $45000\ km$ and $7000\ km$. Determine the following:
  1. Semi-major axis of the elliptical orbit
  2. Orbit eccentricity
  3. Distance between the centre of the Earth and the centre of the elliptical orbit

Problem 2.2

Solution:

1. Semi-major axis of the elliptical orbit a $= \frac{\text{apogee} + \text{perigee}}{2}$

$$= \frac{45\,000 + 7000}{2} = 26\,000\ \text{km}$$

2. Eccentricity $e = \frac{\text{apogee} - \text{perigee}}{2a} = \frac{45\,000 - 7000}{2 \times 26\,000} = \frac{38\,000}{52\,000} = 0.73$

3. Distance between the centre of the Earth and the centre of the ellipse $= ae$

$$= 26\,000 \times 0.73$$

$$= 18\,980\ \text{km}$$

Problem 2.3

- A satellite is moving in an elliptical orbit with the major axis equal to $42000\ km$. If the perigee distance is $8000\ km$, find the apogee and the orbit eccentricity.

Solution:
Major axis = 42 000 km
Also, major axis = apogee + perigee = 42 000 km
Therefore apogee = 42 000 − 8000 = 34 000 km

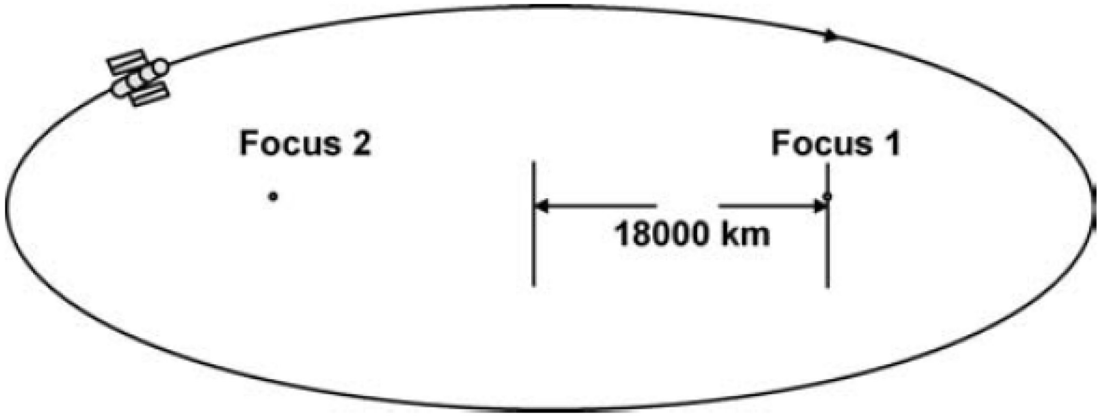
$$\text{Also, eccentricity } e = \frac{\text{apogee} - \text{perigee}}{\text{major axis}}$$

$$= \frac{34\,000 - 8000}{42\,000}$$

$$= \frac{26\,000}{42\,000} = 0.62$$

Problem 2.4

- Determine the apogee and perigee distances if the orbit eccentricity is 0.6.



Solution: If e is the orbit eccentricity and a the semi-major axis of the elliptical orbit, then the distance between the centre of the Earth and the centre of the ellipse is equal to ae.
Therefore $ae = 18\,000$ km
This gives $a = 18\,000/e = 18\,000/0.6 = 30\,000$ km
Apogee distance $= a(1 + e) = 30\,000 \times (1 + 0.6) = 48\,000$ km
Perigee distance $= a(1 - e) = 30\,000 \times (1 - 0.6) = 12\,000$ km

Problem 2.5

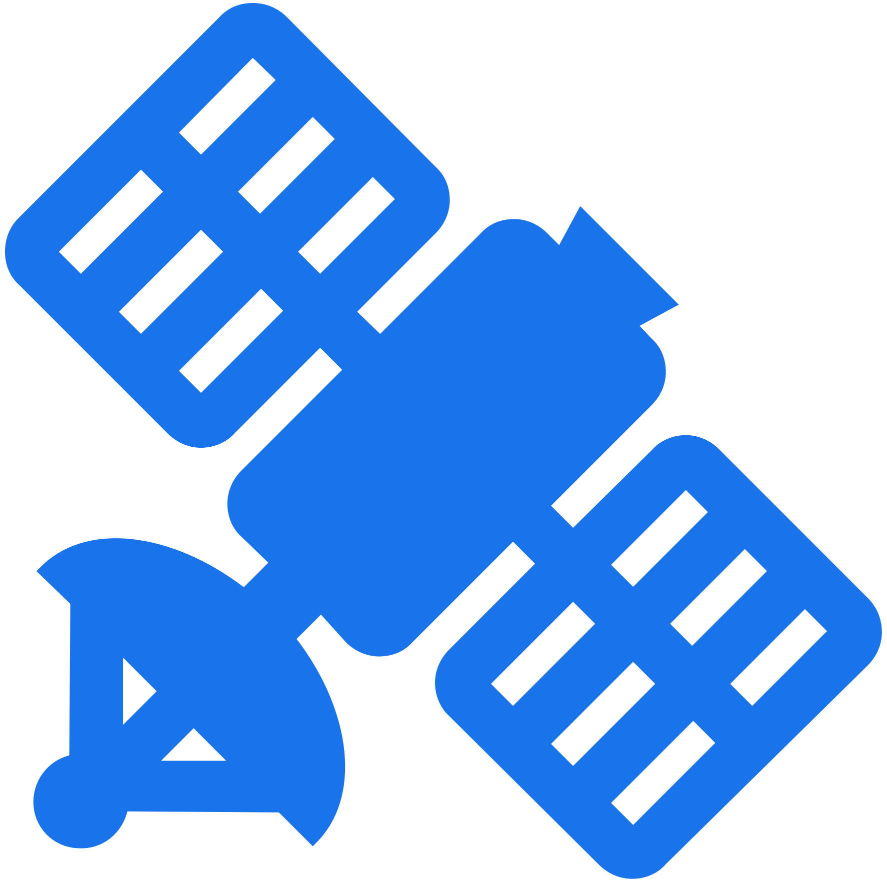
- The difference between the furthest and the closest points in a satellite's elliptical orbit from the surface of the Earth is $30000\ km$ and the sum of the distances is $50000\ km$. If the mean radius of the Earth is considered to be $6400\ km$, determine orbit eccentricity.

Solution: Apogee – Perigee = 30 000 km as the radius of the Earth will cancel in this case

$$\text{Apogee} + \text{Perigee} = 50\,000 + 2 \times 6400 = 62\,800\ \text{km}$$

Orbit eccentricity = (Apogee – Perigee)/(Apogee + Perigee) = 30 000/62 800 = 0.478

Injection Velocity and Resulting Satellite Trajectories



Injection Velocity and Resulting Satellite Trajectories

- **Injection velocity** is the horizontal velocity with which a satellite is injected into space by the launch vehicle with the intention of imparting a specific trajectory to the satellite.
  - This has a direct bearing on the satellite trajectory.
- The phenomenon is best explained in terms of the three cosmic velocities.

First cosmic velocity

- This is the injection velocity at which the apogee and perigee distances are equal, with the result that the satellite orbit is circular.
- The general expression for the velocity of a satellite at the perigee point (V_P), assuming an elliptical orbit, is given by

$$V_P = \sqrt{\left[\left(\frac{2\mu}{r}\right) - \left(\frac{2\mu}{R+r}\right)\right]}$$

where

R = apogee distance
r = perigee distance
$\mu = GM$ = constant

First cosmic velocity

- The first cosmic velocity V_1 is the one at which apogee and perigee distances are equal, i.e. $R = r$, and the orbit is circular.
- The above expression then reduces to

$$V_1 = \sqrt{\left(\frac{\mu}{r}\right)}$$

- Thus, irrespective of the distance r of the satellite from the centre of the Earth, if the injection velocity is equal to the first cosmic velocity, also sometimes called the first orbital velocity, the satellite follows a circular orbit and moves with a uniform velocity equal to (μ/r).

First cosmic velocity

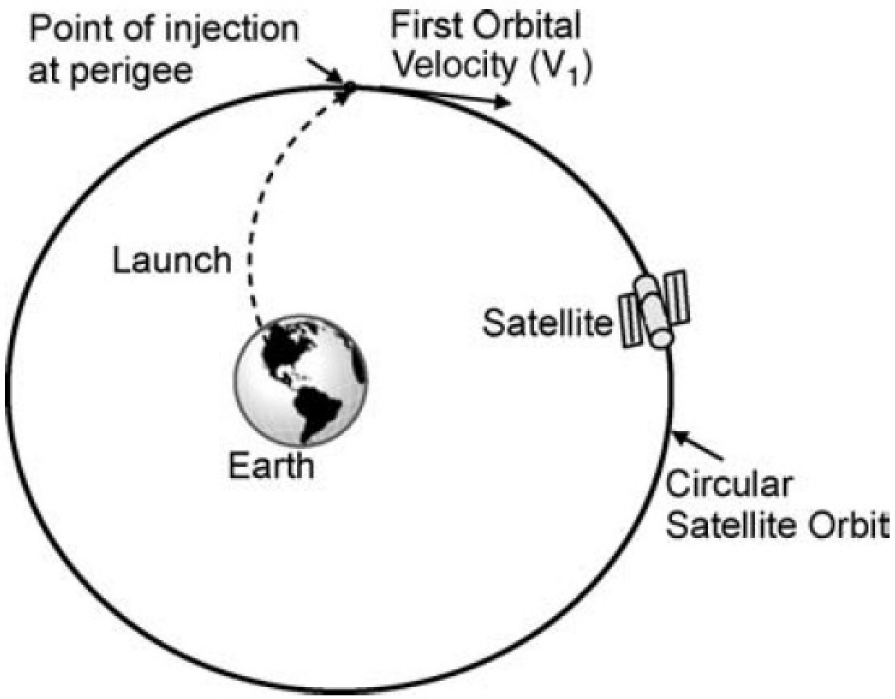


Figure 2.27 Satellite's path where the injection velocity is equal to the first orbital velocity

First cosmic velocity

- If the injection velocity happens to be less than the first cosmic velocity, the satellite follows a ballistic trajectory and falls back to Earth.
- In this case, the orbit is elliptical and the injection point is at the apogee and not the perigee.
- If the perigee lies in the atmosphere or exists only virtually below the surface of the Earth, the satellite accomplishes a ballistic flight and falls back to Earth

First cosmic velocity

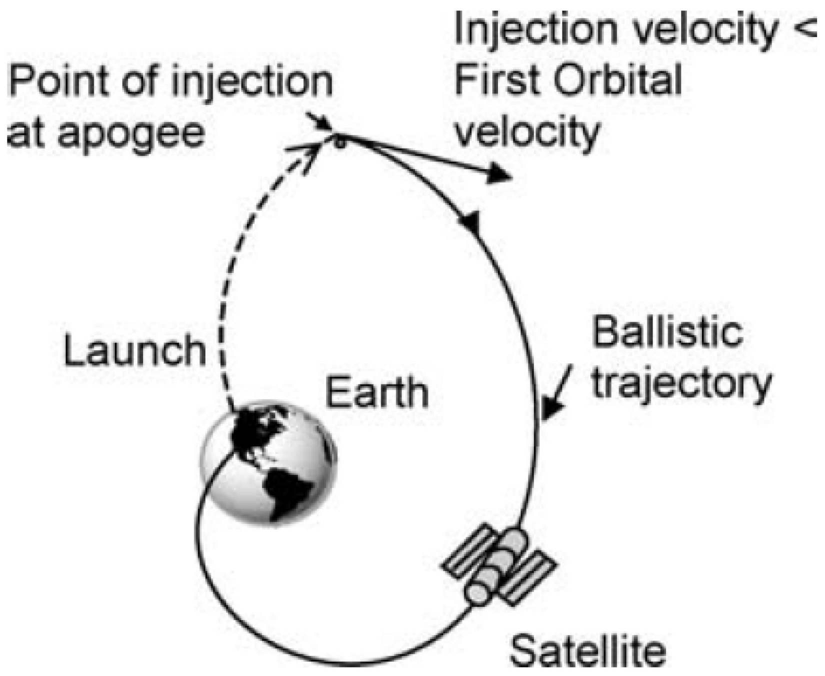


Figure 2.28 Satellite's path where the injection velocity is less than the first orbital velocity

First cosmic velocity

- For injection velocity greater than the first cosmic velocity and less than the second cosmic velocity, i.e. $V > \sqrt{(\mu/r)}$ and $V < \sqrt{(2\mu/r)}$, the orbit is elliptical and eccentric.
- The injection point in this case is the perigee and the apogee distance attained in the resultant elliptical orbit depends upon the injection velocity.
  - The higher the injection velocity, the greater is the apogee distance.
- The apogee distance can also be computed from the known value of injection velocity, which is also the velocity at the perigee point as the perigee coincides with the injection point, and the velocity *v* at any other point in the orbit distant *d* from the centre of the Earth using

$$V_p = \sqrt{\left[(2\mu/r) - \left(\frac{2\mu}{R+r}\right)\right]} = \frac{vd\ \cos\gamma}{r}$$

Second cosmic velocity

- This is the injection velocity at which the apogee distance becomes infinite and the orbit takes the shape of a parabola and the orbit eccentricity is 1.
- It equals √2 times the first cosmic velocity, i.e.

$$V_2 = \sqrt{\left(\frac{2\mu}{r}\right)}$$

- At this velocity, the satellite escapes Earth’s gravitational pull (Also called escape velocity).
- For an injection velocity greater than the second cosmic velocity, the trajectory is hyperbolic within the solar system and the orbit eccentricity is greater than 1.

Second cosmic velocity

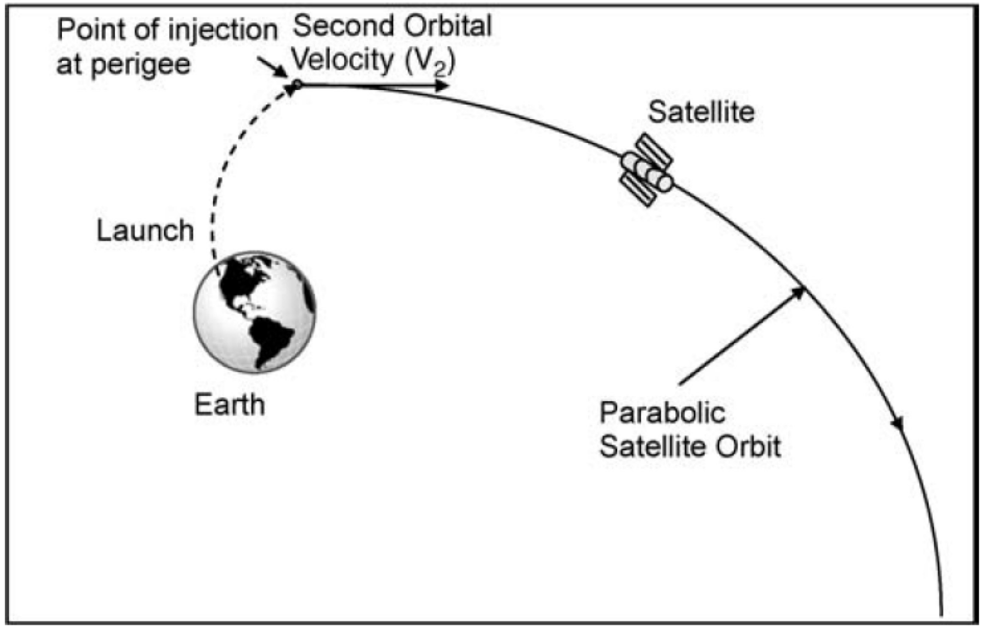


Figure 2.29 Satellite's path where the injection velocity is equal to the second orbital velocity

Third cosmic velocity

- This is the injection velocity at which the satellite succeeds in escaping from the solar system. It is related to the motion of Earth around the sun.
- The third cosmic velocity (V_3) is mathematically expressed as

$$V_3 = \sqrt{\left[\frac{2\mu}{r} - V_t^2(3 - 2\sqrt{2})\right]}$$

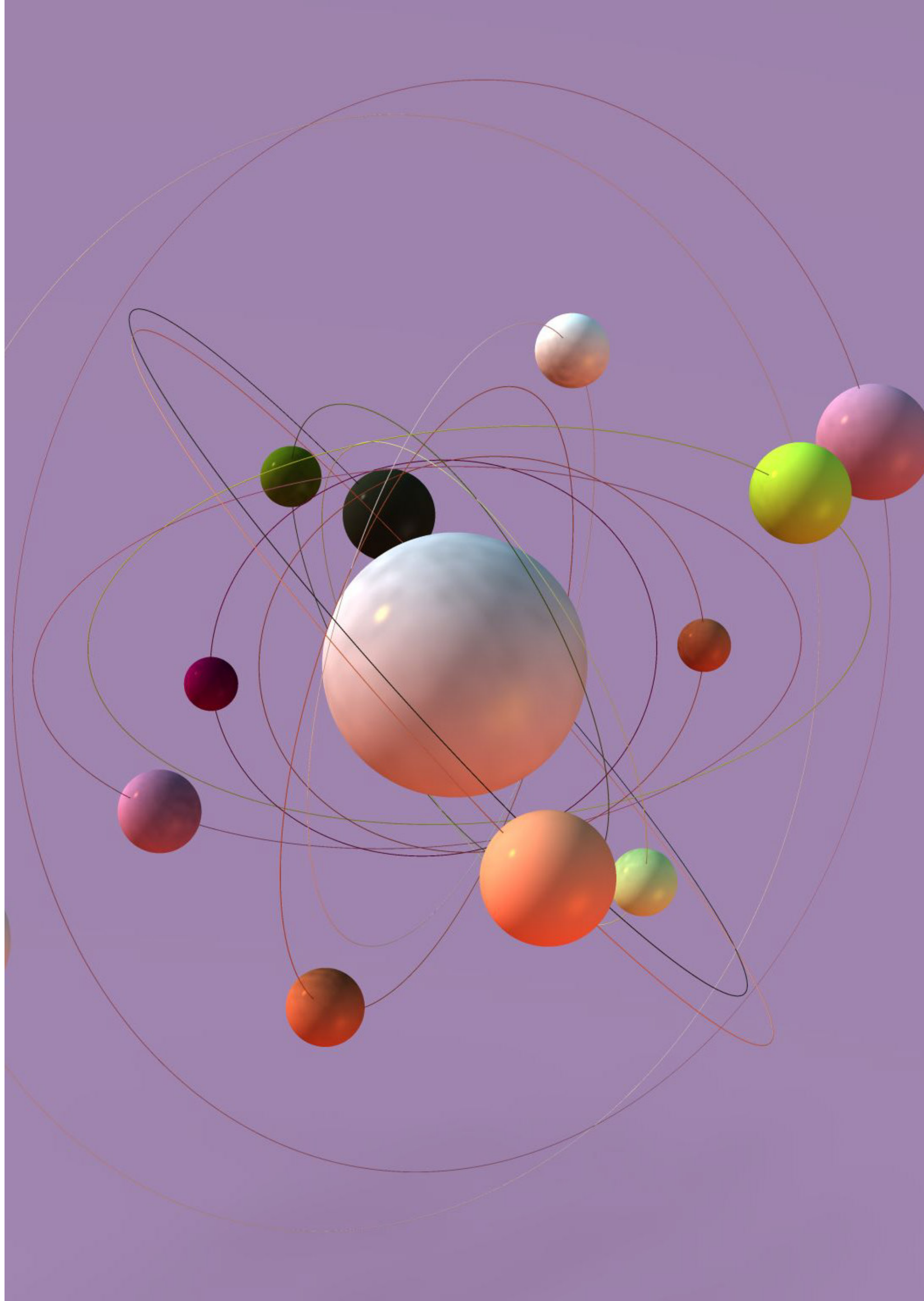
where V_t is the speed of Earth's revolution around the sun.

- For injection velocities beyond the third cosmic velocity, there is a region of hyperbolic flights outside the solar system.

Types of Satellite Orbits

Types of Satellite Orbits

- The satellite orbits can be classified on the basis of:
  1. Orientation of the orbital plane
  2. Eccentricity
  3. Distance from Earth



Orientation of the Orbital Plane

- On this basis, the orbits can be classified as
  - Equatorial orbit
  - Polar orbit
  - Inclined orbit
    - Prograde orbit
    - Retrograde orbit

Equatorial orbit

- In an equatorial orbit, the angle of inclination is zero, i.e. the orbital plane of the satellite coincides with the Earth's equatorial plane.
- A satellite in the equatorial orbit has a latitude of $0°$.

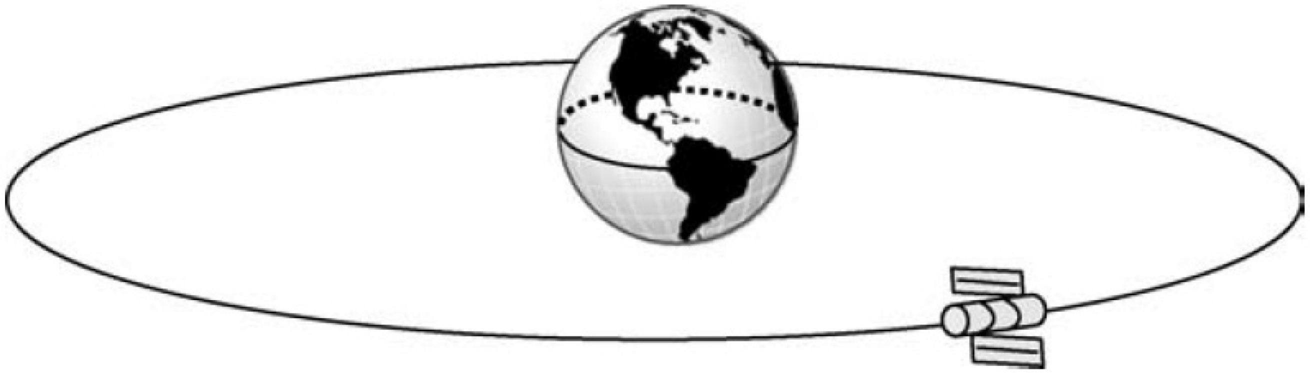


Figure 2.32 Equatorial orbit

Polar orbit

- In a polar orbit, the angle of inclination is equal to $90°$.

Figure 2.33 Polar orbit

Inclined Orbits

- For an angle of inclination between 0◦ and 180◦, the orbit is said to be an inclined orbit.
  - For inclinations between 0◦ and 90◦, the satellite travels in the same direction as the direction of rotation of the Earth. The orbit in this case is referred to as a **direct or prograde orbit**.
  - For inclinations between 90◦ and 180◦, the satellite orbits in a direction opposite to the direction of rotation of the Earth and the orbit in this case is called a **retrograde orbit**.

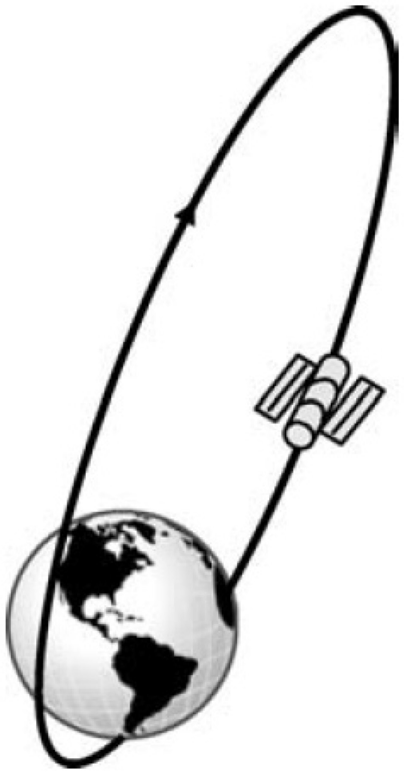


Figure 2.34 Prograde orbit

Figure 2.35 Retrograde orbit

Eccentricity of the Orbit

- On the basis of eccentricity, the orbits are classified as
  - Elliptical Orbits
    - When the orbit eccentricity lies between 0 and 1, the orbit is ***elliptical*** with the centre of the Earth lying at one of the foci of the ellipse.
  - Circular Orbits
    - When the eccentricity is zero, the orbit becomes ***circular***.

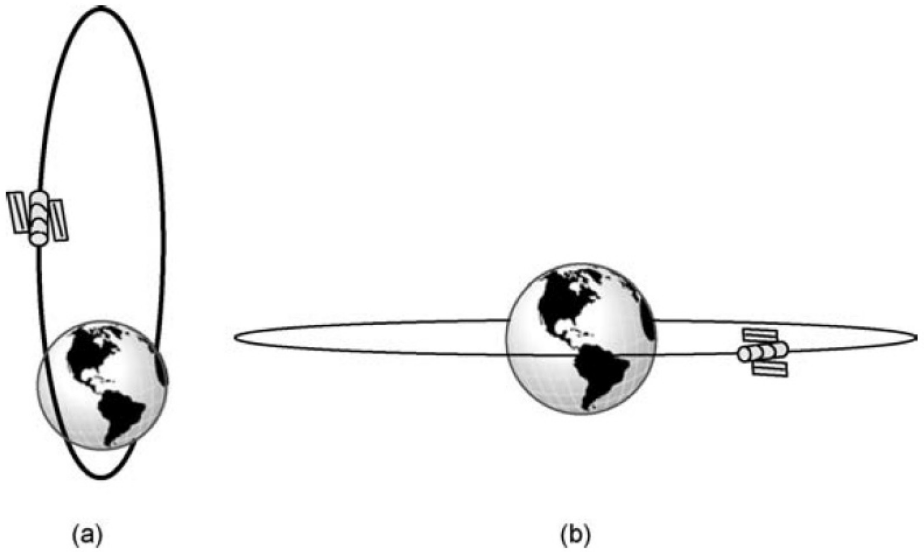


Figure 2.36 (a) Elliptical orbit and (b) circular orbit

Molniya Orbit

- Highly eccentric, inclined and elliptical orbits are used to cover higher latitudes, which are otherwise not covered by geostationary orbits.
- A practical example of this type of orbit is the Molniya orbit.
- It is widely used by Russia and other countries of the former Soviet Union to provide communication services.
- Typical eccentricity and orbit inclination figures for the Molniya orbit are 0.75 and 65° respectively.
- The apogee and perigee points are about $40000\ km$ and $400\ km$ respectively from the surface of the Earth.

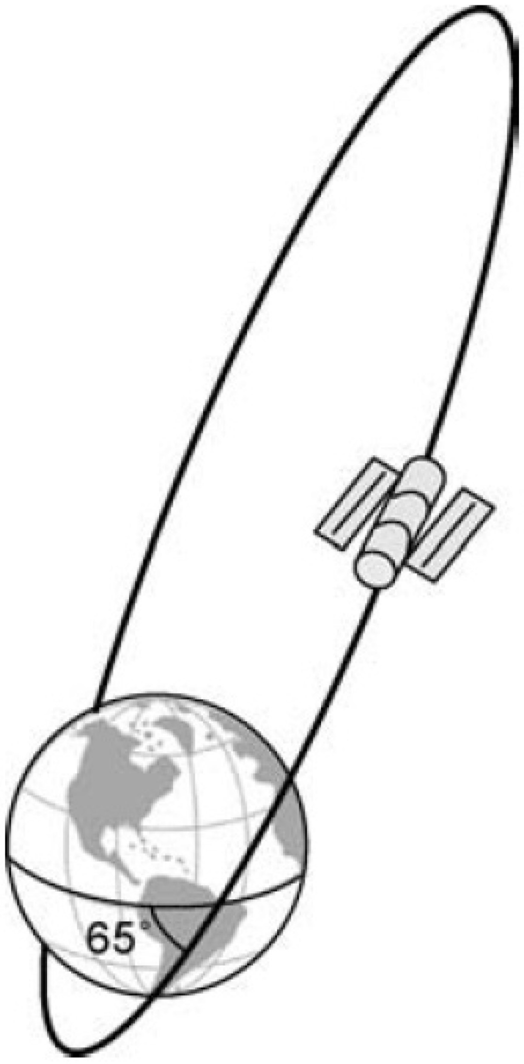


Figure 2.37 Molniya orbit

Molniya Orbit

- The Molniya orbit serves the purpose of a geosynchronous orbit for high latitude regions.
- It is a 12 hour orbit and a satellite in this orbit spends about 8 hours above a particular high latitude station before diving down to a low level perigee at an equally high southern latitude.
- Usually, three satellites at different phases of the same Molniya orbit are capable of providing an uninterrupted service.

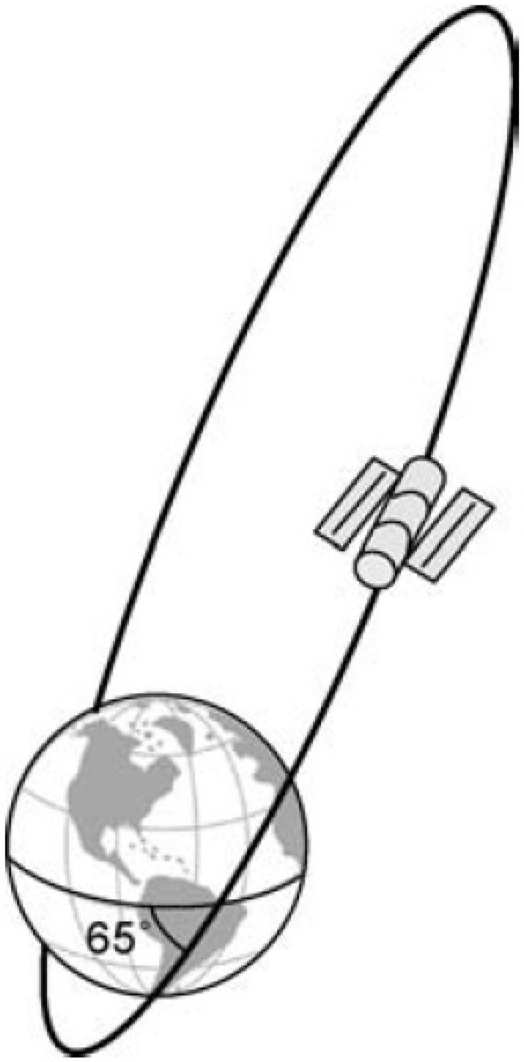


Figure 2.37 Molniya orbit

Distance from Earth

- Depending upon the distance from earth, the orbits are classified as
  - Low Earth orbits (LEOs)
  - Medium Earth orbits (MEOs)
  - Geostationary Earth orbits (GEOs)

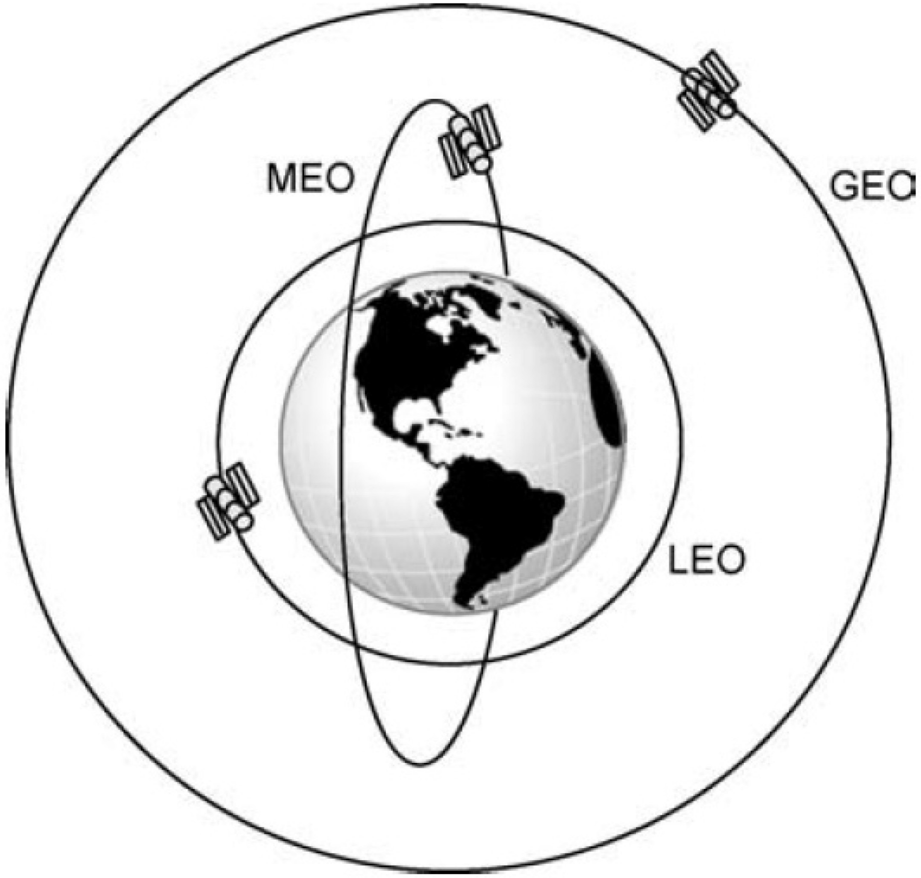


Figure 2.38 LEO, MEO and GEO

Low Earth Orbits (LEOs)

- Satellites in the Low Earth orbit (LEO) circle Earth at a height of around 160 to 500 km above the surface of the Earth.
- These satellites, being closer to the surface of the Earth, have much shorter orbital periods and smaller signal propagation delays.
- A lower propagation delay makes them highly suitable for communication applications.
- Due to lower propagation paths, the power required for signal transmission is also less, with the result that the satellites are of small physical size and are inexpensive to build.
- However, due to a shorter orbital period, of the order of an hour and a half or so, these satellites remain over a particular ground station for a short time.
- Hence, several of these satellites are needed for 24-hour coverage.

Low Earth Orbits (LEOs)

- One important application of LEO satellites for communication is the project Iridium, which is a global communication system conceived by Motorola.
- A total of 66 satellites are arranged in a distributed architecture, with each satellite carrying 1/66 of the total system capacity.
- The system is intended to provide a variety of telecommunication services at the global level.
- The project is named ‘Iridium’ as earlier the constellation was proposed to have 77 satellites and the atomic number of iridium is 77.
- Other applications where LEO satellites can be put to use are surveillance, weather forecasting, remote sensing and scientific studies.

Figure 2.39 Iridium constellation of satellites

Medium Earth Orbits (MEOs)

- Medium Earth Orbit (MEO) satellites orbit at a distance of approximately 10000 to 20000 km above the surface of the Earth.
- They have an orbital period of 6 to 12 hours.
- These satellites stay in sight over a particular region of Earth for a longer time.
- The transmission distance and propagation delays are greater than those for LEO satellites.
- These orbits are generally polar in nature and are mainly used for communication and navigation applications.

Geostationary Earth Orbits (GEOs)

- A geosynchronous Earth orbit is a prograde orbit whose orbital period is equal to Earth's rotational period.
- If such an orbit were in the plane of the equator and circular, it would remain stationary with respect to a given point on the Earth.
- These orbits are referred to as the geostationary Earth orbits (GEOs).
- For the satellite to have such an orbital velocity, it needs to be at a height of about 36000 km, 35786 km to be precise, above the surface of the Earth.

Geostationary Earth Orbits (GEOs)

- In order to remain above the same point on the Earth's surface, a satellite must fulfil the following conditions:
  1. It must have a constant latitude, which is possible only at 0° latitude.
  2. The orbit inclination should be zero.
  3. It should have a constant longitude and thus have a uniform angular velocity, which is possible when the orbit is circular.
  4. The orbital period should be equal to 23 hours 56 minutes, which implies that the satellite must orbit at a height of 35786 km above the surface of the Earth.
  5. The satellite motion must be from west to east.

Geostationary Earth Orbits (GEOs)

- In the case where these conditions are fulfilled, then as the satellite moves from a position O_1 to O_2 in its orbit, a point vertically below on the equator moves with the same angular velocity and moves from E_1 to E_2.
- Satellites in geostationary orbits play an essential role in relaying communication and TV broadcast signals around the globe.
- They also perform meteorological and military surveillance functions very effectively.

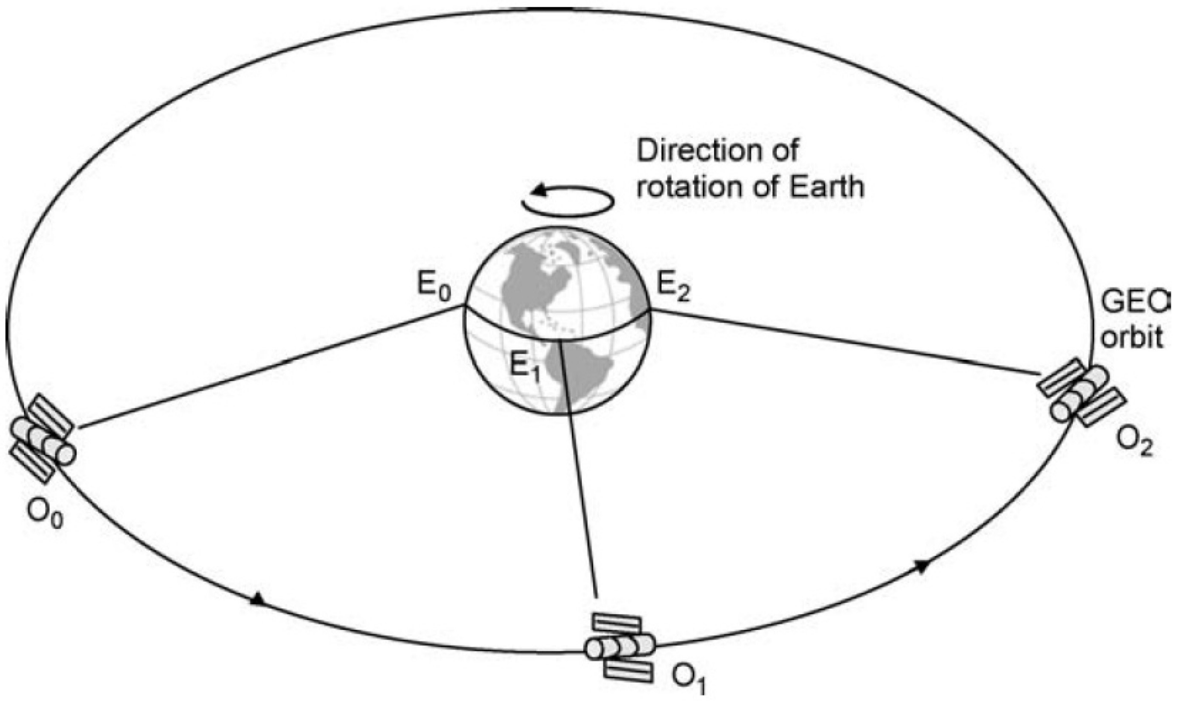


Figure 2.40 GEO satellites appear stationary wrt to a point on Earth

Sun-synchronous Orbit

- A ***sun-synchronous orbit***, also known as a ***helio-synchronous orbit***, is one that lies in a plane that maintains a fixed angle with respect to the Earth–sun direction.
- In other words, the orbital plane has a fixed orientation with respect to the Earth–sun direction and the angle between the orbital plane and the Earth–sun line remains constant throughout the year.
- Satellites in sun-synchronous orbits are particularly suited to applications like passive remote sensing, meteorological, military reconnaissance and atmospheric studies.

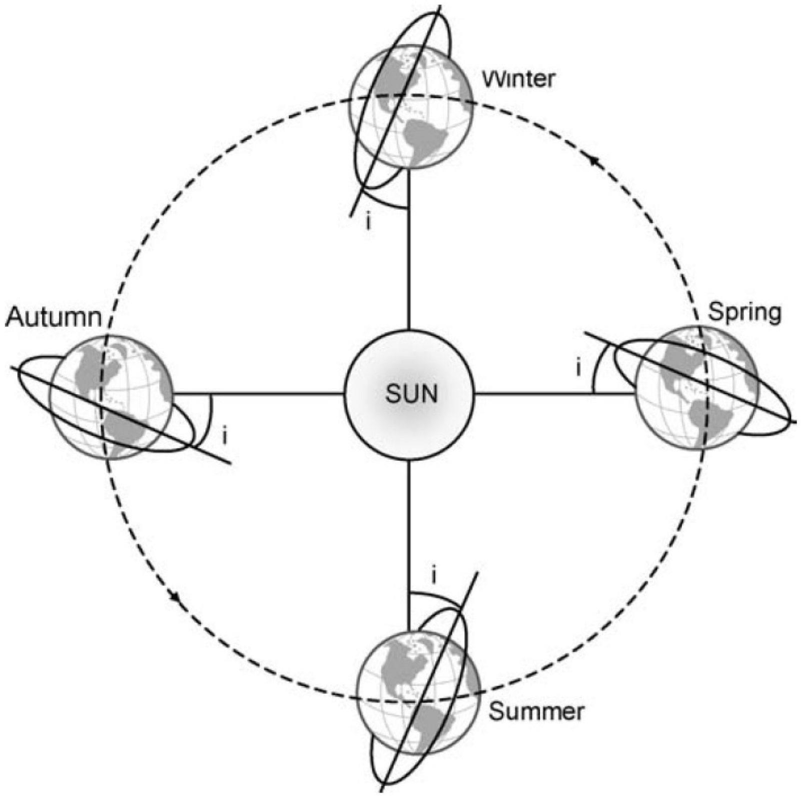


Figure 2.41 Sun-synchronous orbit

Sun-synchronous Orbit

- As a result of this property, sun-synchronous orbits ensure that:
  1. The satellite passes over a given location on Earth every time at the same local solar time, thereby guaranteeing almost the same illumination conditions, varying only with seasons.
  2. The satellite ensures coverage of the whole surface of the Earth, being quasi-polar in nature.

Sun-synchronous Orbit

- Every time a sun-synchronous satellite completes one revolution around Earth, it traverses a thin strip on the surface of the Earth.
  - During the next revolution it traverses another strip shifted westwards and the process of shift continues with successive revolutions.
- Depending upon the orbital parameters and speed of rotation of Earth, after making a certain number of revolutions around Earth, it comes back close to the first strip that it had traversed.
  - It may not exactly overlap the first strip, as the mean distance between the two strips, called the tracking interval, may not be an integral multiple of the equatorial perimeter.
- However, the number of revolutions required before the satellite repeats the same strip sequence can certainly be calculated.
  - This is called one complete ***orbital cycle***, which is basically the time that elapses before the satellite revisits a given location in the same direction.
- To be more precise, orbital cycle means the whole number of orbital revolutions that a satellite must describe in order to be once again flying in the same direction over the same point on the Earth's surface.

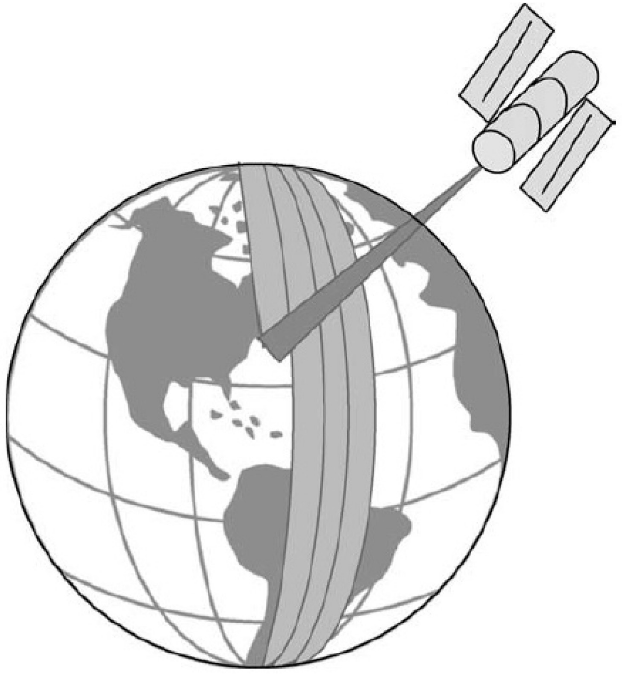


Figure 2.42 Earth coverage of sun-synchronous satellites

Orbital Perturbations

Orbital Perturbations

- The satellite, once placed in its orbit, experiences various perturbing torques that cause variations in its orbital parameters with time.
- These include gravitational forces from other bodies like solar and lunar attraction, magnetic field interaction, solar radiation pressure, asymmetry of Earth's gravitational field etc.
- Due to these factors, the satellite orbit tends to drift and its orientation also changes and hence the true orbit of the satellite is different from that defined using Kepler's laws.

Orbital Perturbations

- The satellite's position thus needs to be controlled both in the east–west as well as the north–south directions.
  - The east–west location needs to be maintained to prevent radio frequency (RF) interference from neighbouring satellites.
    - It may be mentioned here that in the case of a geostationary satellite, a 1° drift in the east or west direction is equivalent to a drift of about 735 km along the orbit.
  - The north–south orientation has to be maintained to have proper satellite inclination.

Orbital Perturbations

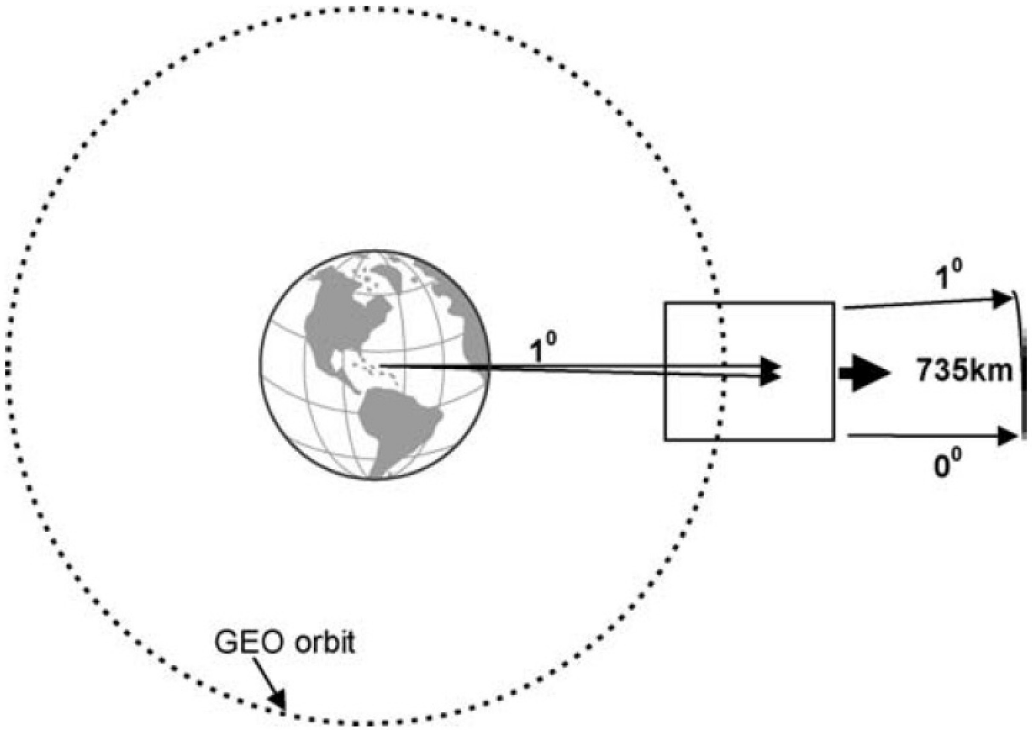


Figure 3.27 Drift of a geostationary satellite

Orbital Perturbations

- Earth is not a perfect sphere; flattened at poles.
  - Equatorial diameter ≈ 20–40 km greater than polar diameter.
  - Equatorial radius not constant.
  - Average density of Earth is non-uniform.
  - Result: Non-uniform gravitational field around Earth.
- Variations in Earth's gravity affect satellites differently.
  - LEO satellites: Rapid movement averages out gravitational variations.
  - Geostationary satellites: More affected due to fixed position relative to Earth.
    - Perturbing forces cause acceleration/deceleration depending on longitude.
    - Forces vary with satellite's position over Earth's surface.

Orbital Perturbations

- In addition to the variation in the gravitational field of the Earth, the satellite is also subjected to the gravitational pulls of the sun and the moon.
  - Sun's gravity: Earth's orbit inclined by ~7° to sun's equator.
  - Earth tilted ~23° from normal to ecliptic.
  - Moon's orbit inclined ~5° to Earth's equator.
- These create out-of-plane forces altering orbital inclination.
  - In LEO, atmospheric drag dominates; gravitational variations less significant.
  - In GEO, gravitational effects from Earth, Sun, and Moon are major factors.

Orbital Perturbations

- As the perturbed orbit is not an ellipse anymore, the satellite does not return to the same point in space after one revolution.
- The time elapsed between the successive perigee passages is referred to as anomalistic period.
- The anomalistic period (t_A) is given by

$$t_A = \frac{2\pi}{\omega_{\text{mod}}}$$

Where,

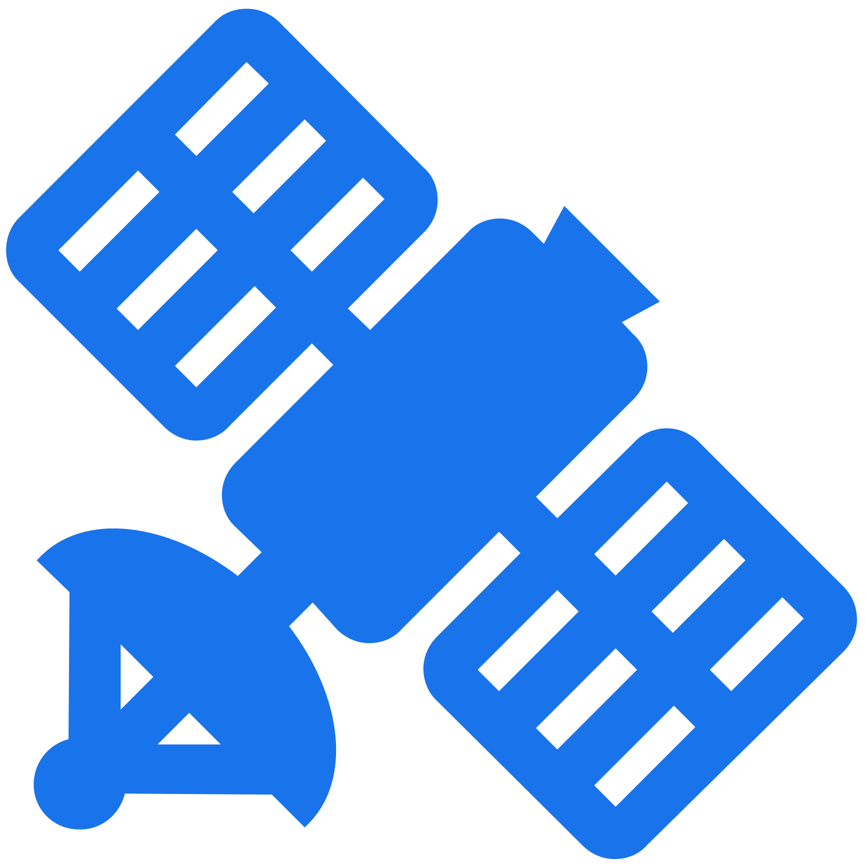
$$\omega_{\text{mod}} = \omega_0 \left[1 + \frac{K(1 - 1.5 \sin^2 i)}{a^2(1 - e^2)^{3/2}} \right]$$

ω_0 is the angular velocity for spherical Earth, $K = 66\,063.1704 \text{km}^2$, a is the semi-major axis, e is the eccentricity and $i = \cos^{-1} W_Z$, W_Z is the Z axis component of the orbit normal.

Orbital Perturbations

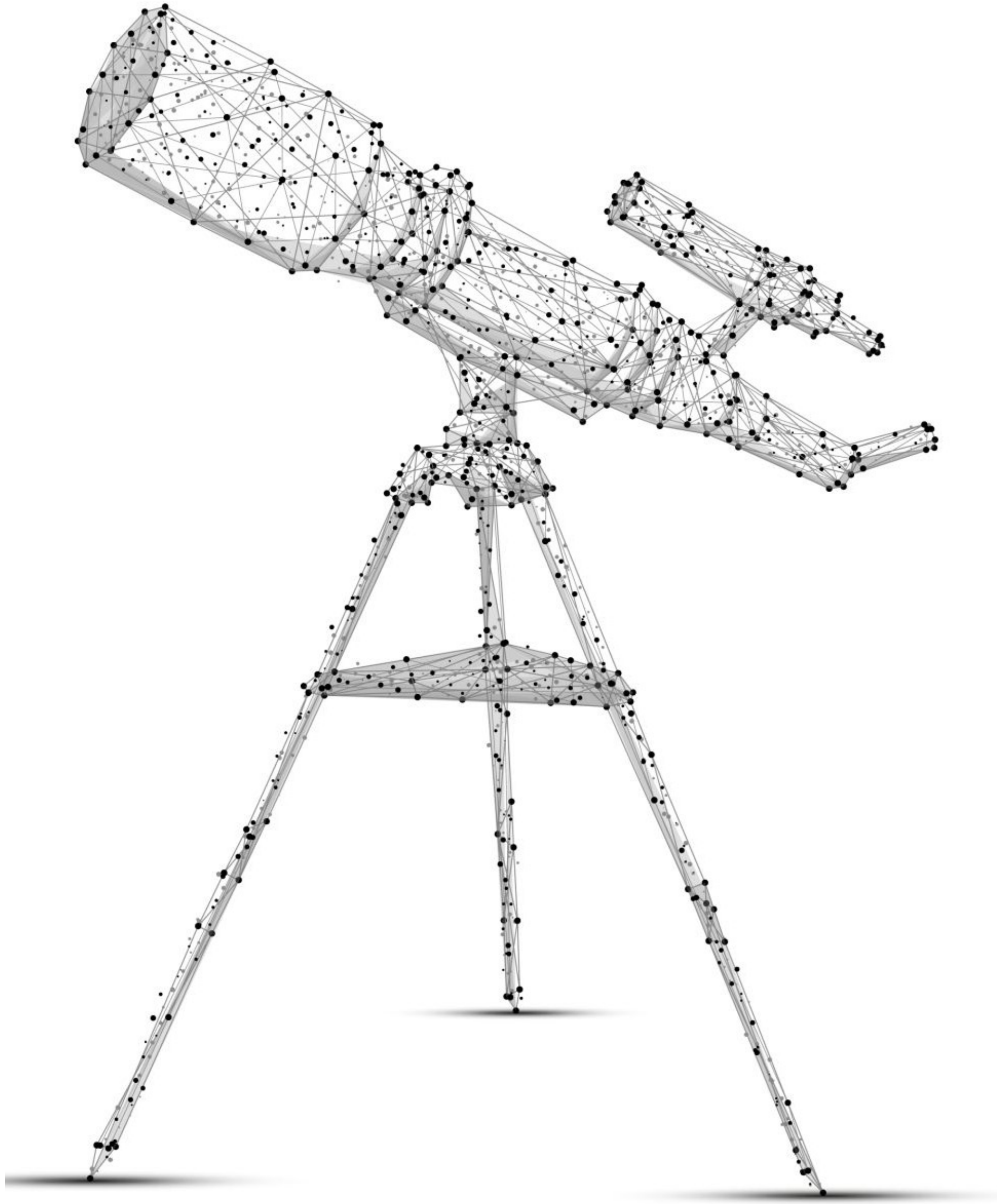
- The attitude and orbit control system maintains the satellite's position and its orientation and keeps the antenna pointed correctly in the desired direction (bore-sighted to the center of the coverage area of the satellite).
- The orbit control is performed by firing thrusters in the desired direction or by releasing jets of gas.
  - It is also referred to as station keeping.
- Thrusters and gas jets are used to correct the longitudinal drifts (in-plane changes) and the inclination changes (out-of-plane changes).
- Generally, a different set of thrusters or gas jets is used for north-south and east-west manoeuvres.
  - North-South manoeuvre:
    - Corrects longitudinal drift.
    - Requires larger velocity increment.
  - East-West manoeuvre:
    - Corrects inclination changes.
    - Requires smaller velocity increment.

Satellite Stabilization



Satellite Stabilization

- Commonly employed techniques for satellite attitude control include:
  1. Spin stabilization
  2. Three-axis or body stabilization



Spin Stabilization

- In a spin-stabilized satellite, the satellite body is spun at a rate between 30 and 100 rpm about an axis perpendicular to the orbital plane.
- Like a spinning top, the rotating body offers inertial stiffness, which prevents the satellite from drifting from its desired orientation.
- Spin-stabilized satellites are generally cylindrical in shape.
- For stability, the satellite should be spun about its major axis, having a maximum moment of inertia.
- To maintain stability, the moment of inertia about the desired spin axis should at least be 10% greater than the moment of inertia about the transverse axis.

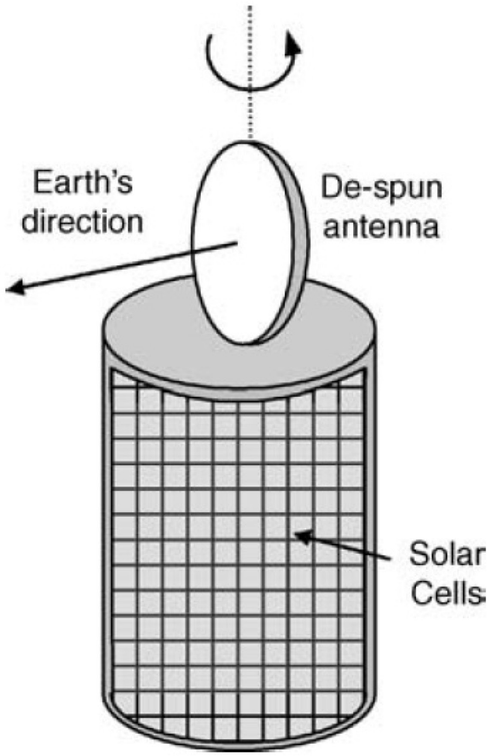


Figure 3.28 Spin stabilized satellite

Spin Stabilization

- There are two types of spinning configurations employed in spin-stabilized satellites.
  - Simple Spinner Configuration
    - Payload & subsystems in **spinning section**.
    - Antenna & feed in **de-spun platform**.
    - De-spun platform rotates opposite to spinning body.
  - Dual Spinner Configuration
    - Payload, antenna & feed in **de-spun platform**.
    - Other subsystems in **spinning body**.
    - Common in modern spin-stabilized satellites.
- Examples of Spin-Stabilized Satellites
  - Intelsat-1 to Intelsat-4, Intelsat-6, TIROS-1

Intelsat-4

TIROS-1

Three-axis or Body Stabilization

- In the case of three-axis stabilization, also known as body stabilization, the stabilization is achieved by controlling the movement of the satellite along the three axes, i.e. *yaw, pitch* and *roll*, with respect to a reference.

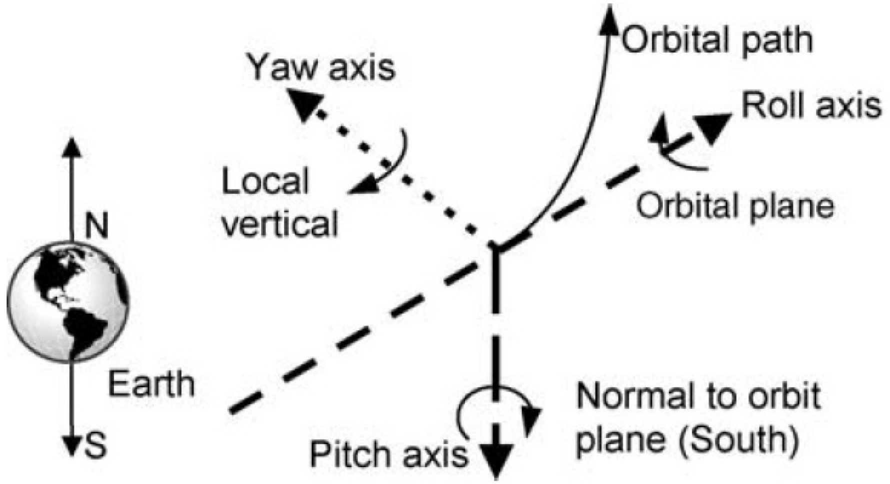


Figure 3.30 Three-axis stabilization

Three-axis or Body Stabilization

- The system uses reaction wheels or momentum wheels to correct orbit perturbations.
- The stability of the three-axis system is provided by the active control system, which applies small corrective forces on the wheels to correct the undesirable changes in the satellite orbit.

Three-axis or Body Stabilization

- Most three-axis stabilized satellites use momentum wheels.
- The basic control technique used here is to speed up or slow down the momentum wheel depending upon the direction in which the satellite is perturbed.
- The satellite rotates in a direction opposite to that of speed change of the wheel.
  - For example, an increase in speed of the wheel in the clockwise direction will make the satellite to rotate in a counterclockwise direction.
- The momentum wheels rotate in one direction and can be twisted by a gimbal motor to provide the required dynamic force on the satellite.

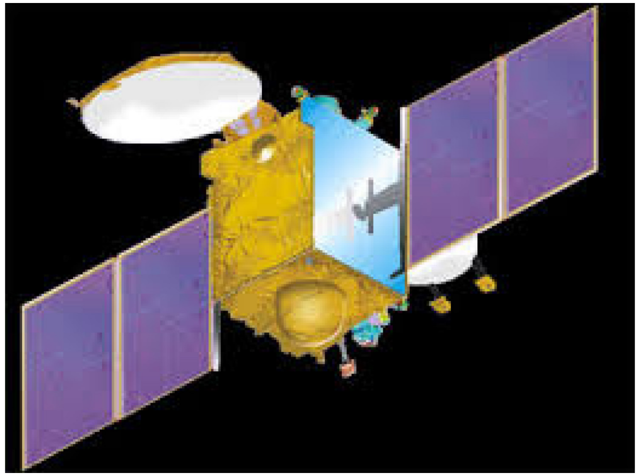
Three-axis or Body Stabilization

- An alternative approach is to use reaction wheels.
- Three reaction wheels are used, one for each axis.
- They can be rotated in either direction depending upon the active correction force.
- The satellite body is generally box shaped for three-axis stabilized satellites.
- Antennae are mounted on the Earth-facing side and on the lateral sides adjacent to it.
- These satellites use flat solar panels mounted above and below the satellite body in such a way that they always point towards the sun, which is an obvious requirement.

Three-axis or Body Stabilization

- Some popular satellites belonging to the category of three-axis stabilized satellites include Intelsat-5, Intelsat-7, Intelsat-8, GOES-8, GOES-9, TIROS-N and the INSAT series of satellites.

Intelsat-5



INSAT-4C

Comparison between Spin-stabilized and Three-axis Stabilized Satellites

1. In comparison to spin-stabilized satellites, three-axis stabilized satellites have more power generation capability and more additional mounting area available for complex antennae structures.
2. Spin-stabilized satellites are simpler in design and less expensive than three-axis stabilized satellites.
3. Three-axis stabilized satellites have the disadvantage that the extendible solar array used in these satellites are unable to provide power when the satellite is in the transfer orbit, as the array is still stored inside the satellite during this time.

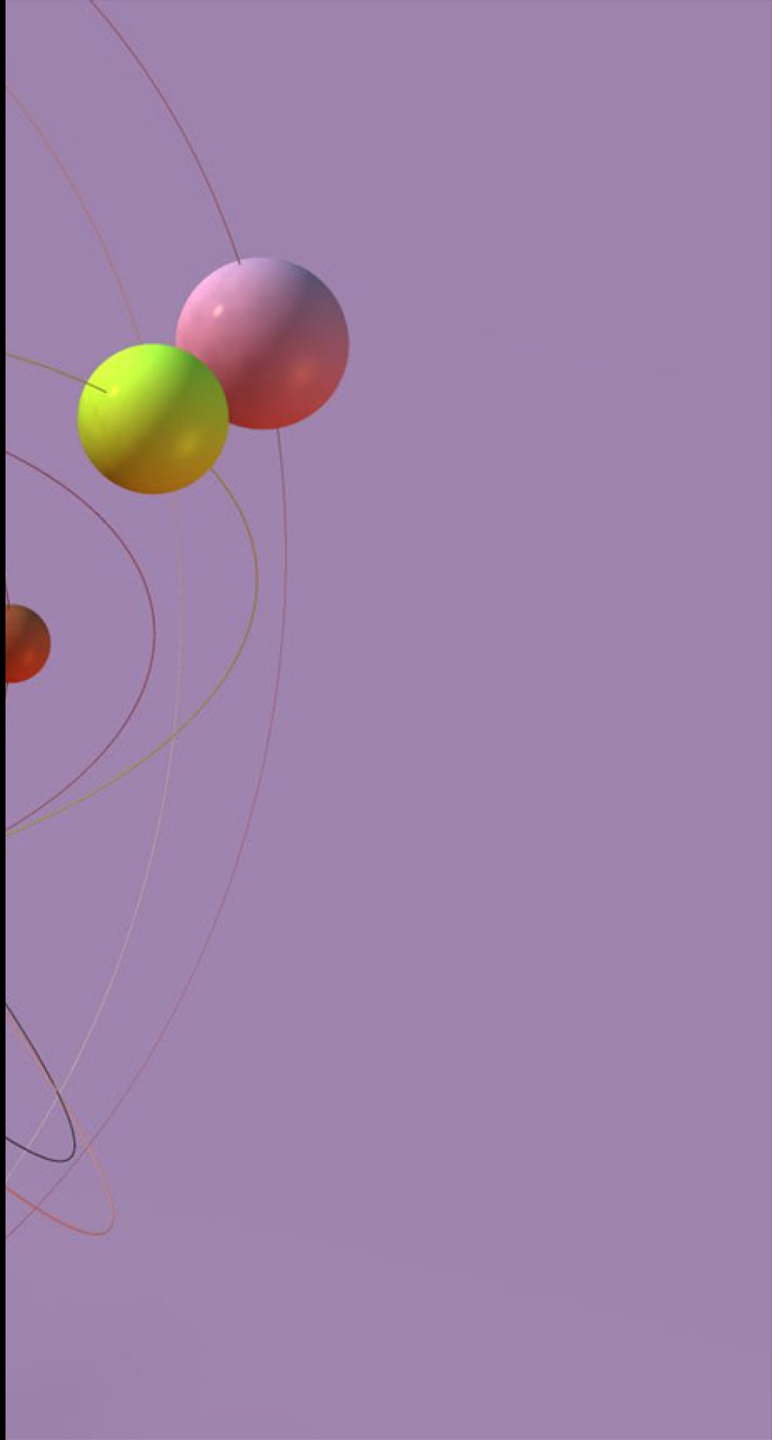
Station Keeping

- *Station keeping* is the process of maintenance of the satellite's orbit against different factors that cause temporal drift.
- Causes of Orbital Drift
  - Gravitational perturbations (Sun & Moon)
  - Solar radiation pressure
  - Earth's imperfect spherical shape
- Methods of Orbital Adjustment
  - Releasing jets of gas
  - Firing small onboard rockets (thrusters)

Station Keeping

- In case of Spin-Stabilized Satellites
  - North–South control → Thrusters parallel to spin axis (continuous mode)
  - East–West control → Thrusters perpendicular to spin axis
- In case of Three-Axis Stabilized Satellites
  - Station keeping by firing thrusters in east–west or north–south directions (continuous mode)

Orbital Effects on Satellite's Performance



Orbital Effects on Satellite's Performance

- The motion of the satellite has significant effects on its performance.
- These include
  - Doppler shift
  - Variation in the orbital distance
  - Solar eclipse
  - Sun's transit outrage

Doppler Shift

- The geostationary satellites appear stationary with respect to an Earth station terminal whereas in the case of satellites orbiting in low Earth orbits, the satellite is in relative motion with respect to the terminal.
  - However, in the case of geostationary satellites also there are some variations between the satellite and the Earth station terminal.
- As the satellite is moving with respect to the Earth station terminal, the frequency of the satellite transmitter also varies with respect to the receiver on the Earth station terminal.
- If the frequency transmitted by the satellite is f_T , then the received frequency f_R is given by

$$\left(\frac{f_R - f_T}{f_T}\right) = \left(\frac{\Delta f}{f_T}\right) = \left(\frac{v_T}{v_P}\right)$$

Where,
v_T is the component of the satellite transmitter velocity vector directed towards the Earth station receiver
v_P is the phase velocity of light in free space (3×10^8 m/s)

Variation in the Orbital Distance

- Variation in the orbital distance results in variation in the range between the satellite and the Earth station terminal.
- If a Time Division Multiple Access (TDMA) scheme is employed by the satellite, the timing of the frames within the TDMA bursts should be worked out carefully so that the user terminals receive the correct data at the correct time.
- Range variations are more predominant in low and medium Earth orbiting satellites as compared to the geostationary satellites.

Solar Eclipse

- There are times when the satellites do not receive solar radiation due to obstruction from a celestial body.
- Satellites rely on onboard batteries during these periods.
- Battery design ensures continuous power supply during eclipse periods.
- Ground control routines before eclipse:
  - Discharge batteries close to max depth.
  - Fully recharge before eclipse.
- Sudden entry/exit from shadow causes thermal stresses.
  - Satellites are designed to withstand these stresses.

Sun Transit Outrage

- There are times when the satellite passes directly between the sun and the Earth.
- The Earth station antenna will receive signals from the satellite as well as the microwave radiation emitted by the sun
  - The sun is a source of radiation with an equivalent temperature varying between 6000K to 11000K.
- This might cause temporary outrage if the magnitude of the solar radiation exceeds the fade margin of the receiver.
- The traffic of the satellite may be shifted to other satellites during such periods.

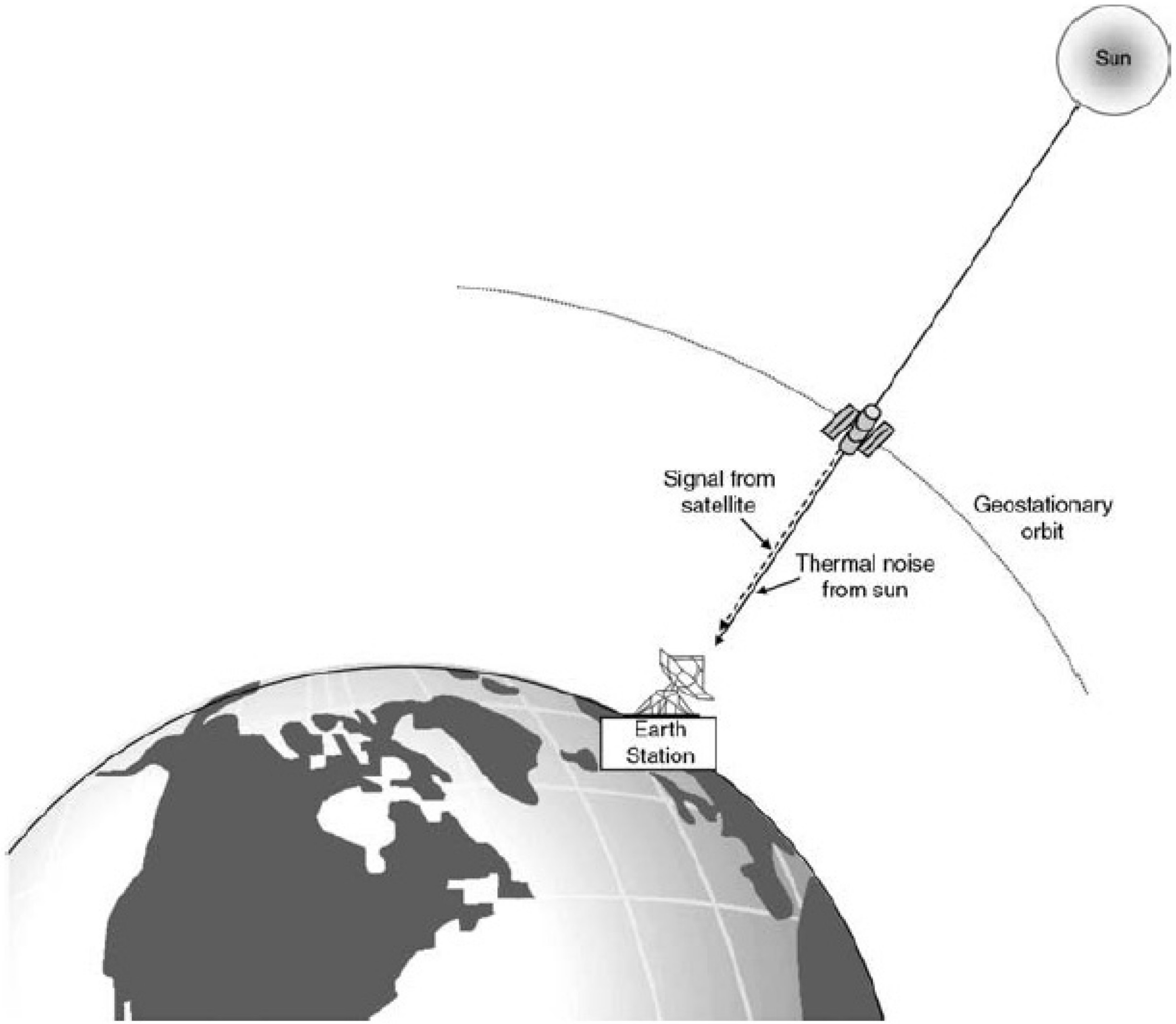


Figure 3.32 Sun outrage conditions

Eclipses

Eclipses

- With reference to satellites, an eclipse is said to occur when the sunlight fails to reach the satellite's solar panel due to an obstruction from a celestial body.
- There are two types of eclipses:
  - Solar eclipse
  - Lunar eclipse

Eclipses

- Solar eclipse occurs when the satellite comes in the shadow of the Earth.

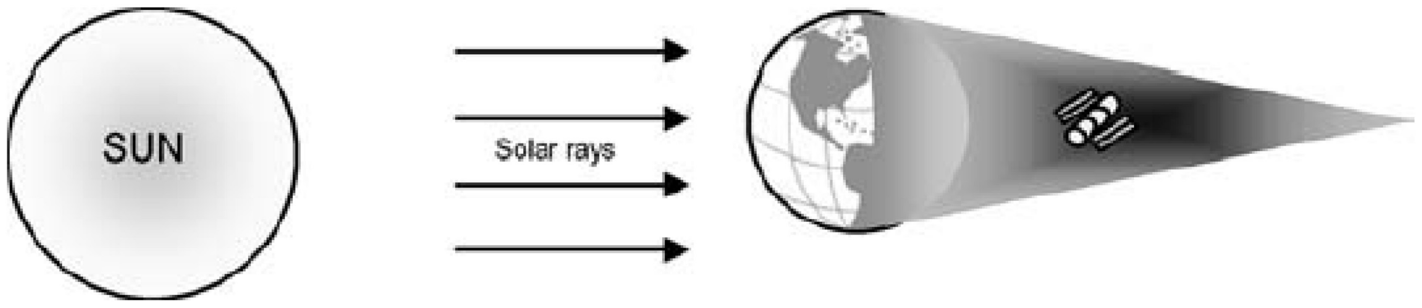


Figure 3.33 Solar eclipse

Eclipses

- The Earth's shadow has two distinct parts:
  - **Umbra**: The dark, central region of the shadow. If a satellite passes through this, it experiences a **total eclipse** and receives no sunlight at all.
  - **Penumbra**: The less dark, outer region. A satellite in the penumbra receives very little sunlight.

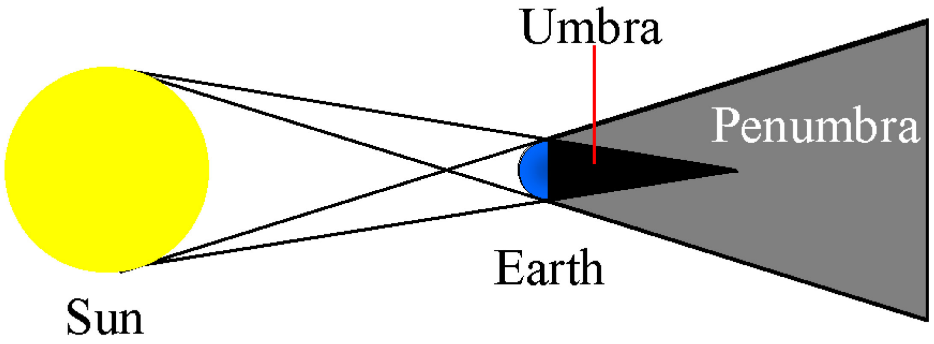


Figure 3.34 Umbra and penumbra

Eclipses

- The key reason for these eclipses is the **23.5° tilt** between two planes:
  - **The Ecliptic Plane**: The plane of the Earth's orbit around the Sun.
  - **The Equatorial Plane**: The plane above the Earth's equator, where geostationary satellites orbit.
- Because of this tilt, the satellite's path only intersects with Earth's shadow at specific times of the year.

Eclipses

- For geostationary satellites, eclipses occur during two 42-day periods during spring and autumn every year.
- These periods are centered around the equinoxes.
  - **Spring Equinox**: 20-21 March
  - **Autumn Equinox**: 22-23 September
- During these times, the Sun, Earth, and satellites are aligned at midnight local time and the satellite spends about 72 minutes in total darkness.

Eclipses

- From 21 days before and 21 days after the equinoxes, the satellite crosses the umbral cone each day for some time, thereby receiving only a part of solar light for that time.
- During the rest of the year, the 23.5° tilt causes the satellite's orbit to pass either above or below the Earth's shadow.
  - **Summer Solstice (20–21 June)**: The satellite is at its maximum distance *above* the umbral cone.
  - **Winter Solstice (21–22 December)**: The satellite is at its maximum distance *below* the umbral cone.

Eclipses

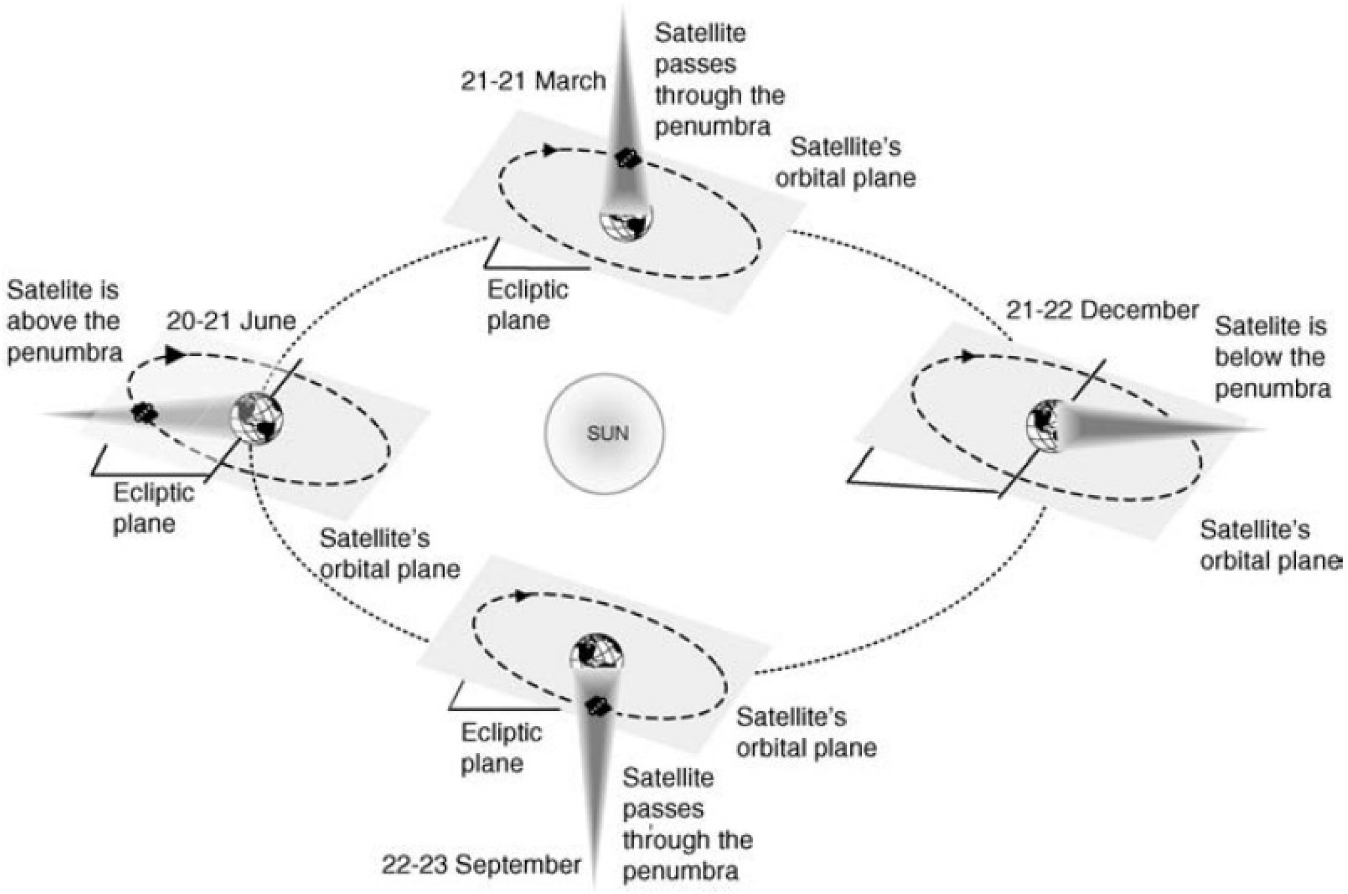


Figure 3.35 Positions of the geostationary satellite during the equinoxes and solstices

Eclipses

- The duration of an eclipse increases from zero to about 72 minutes starting 21 days before the equinox and then decreases from 72 minutes to zero during 21 days following the equinox.

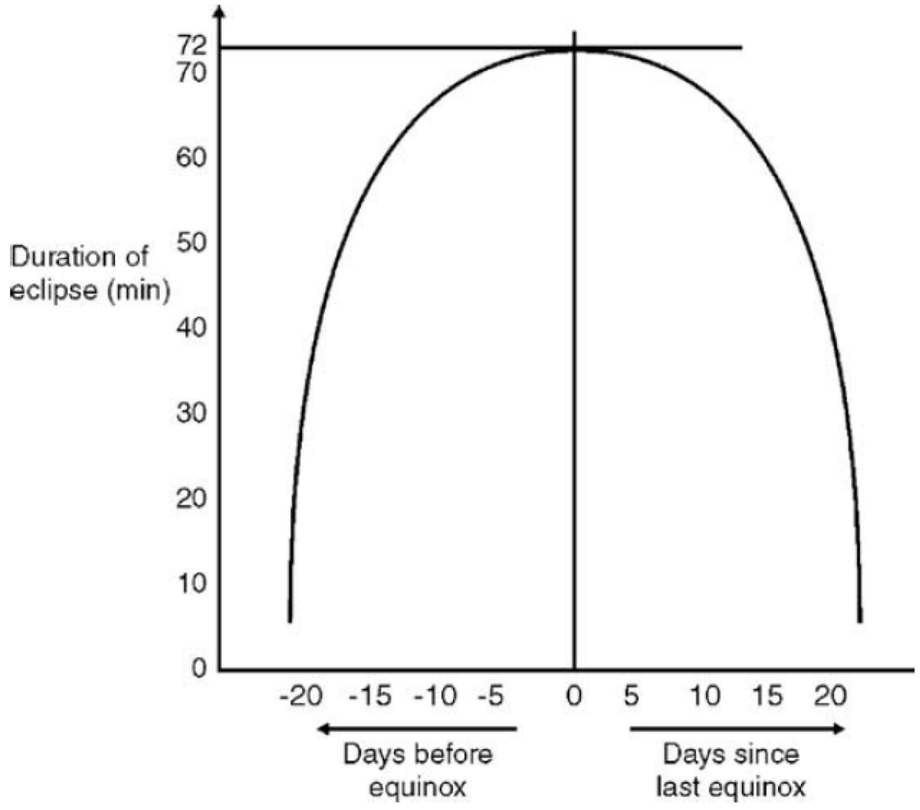


Figure 3.36 Duration of the eclipse before and after the equinox

Eclipses

- While a solar eclipse takes place, the failure of sunlight to reach the satellite interrupts the battery recharging process.
- The satellite is depleted of its electrical power capacity.
- It does not significantly affect low power satellites, which can usually continue their operation with back-up power.
- The high power satellites, however, shut down for all but essential services.

Eclipses

- Lunar eclipse occurs when the moon's shadow passes across the satellite.
- This is much less common and occurs once in 29 years.

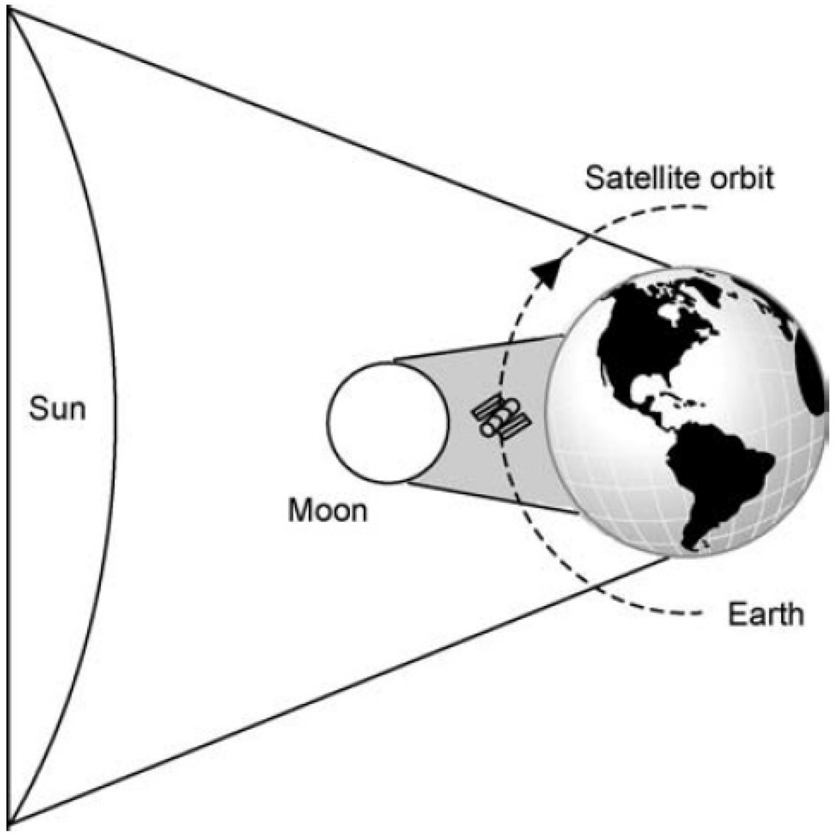


Figure 3.37 Lunar eclipse

Look Angles of a Satellite

Look Angles of a Satellite

- The look angles of a satellite refer to the coordinates to which an Earth station must be pointed in order to communicate with the satellite.
    - Azimuth angle
    - Elevation angle
- In order to determine the look angles of a satellite, its precise location should be known.
    - The location of a satellite is very often determined by the position of the sub-satellite point.
- The *sub-satellite point* is the location on the surface of the Earth that lies directly between the satellite and the centre of the Earth.
    - To an observer on the sub-satellite point, the satellite will appear to be directly overhead

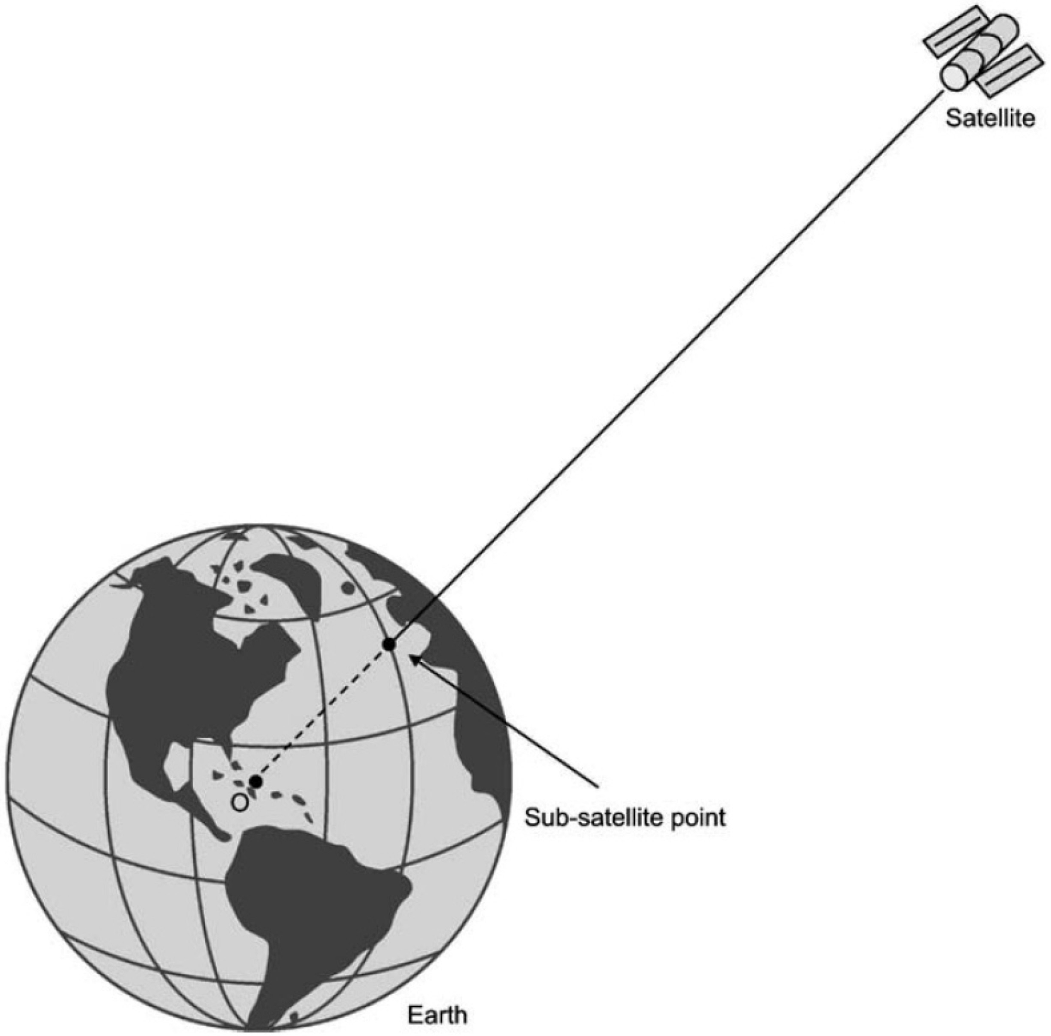


Figure 3.38 Sub-satellite point

Azimuth Angle

- The azimuth angle *A* of an Earth station is defined as the angle produced by the line of intersection of the local horizontal plane and the plane passing through the Earth station, the satellite and the centre of the Earth with the true north.

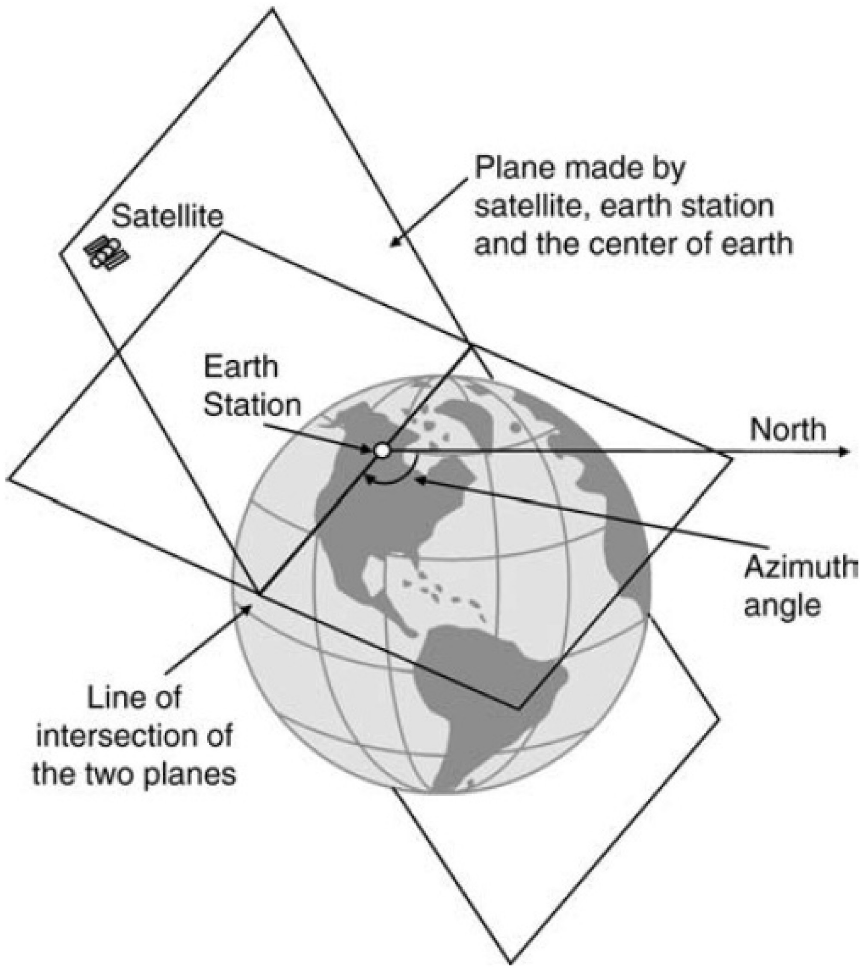


Figure 3.39 Azimuth angle

Azimuth Angle

- Depending upon the location of the Earth station and the sub-satellite point, the azimuth angle can be computed as follows:
- Earth station in the northern hemisphere:

$A = 180^\circ - A'$. when the Earth station is to the west of the satellite

$A = 180^\circ + A'$. when the Earth station is to the east of the satellite

- Earth station in the southern hemisphere:

$A = A' \ldots$ when the Earth station is to the west of the satellite

$A = 360^\circ - A' \ldots$ when the Earth station is to the east of the satellite

where A' can be computed from

$$A' = \tan^{-1}\left(\frac{\tan|\theta_s - \theta_L|}{\sin\theta_l}\right)$$

where

θ_s = satellite longitude

θ_L = Earth station longitude

θ_l = Earth station latitude

Elevation Angle

- The Earth station elevation angle *E* is the angle between the line of intersection of the local horizontal plane and the plane passing through the Earth station, the satellite and the centre of the Earth with the line joining the Earth station and the satellite.

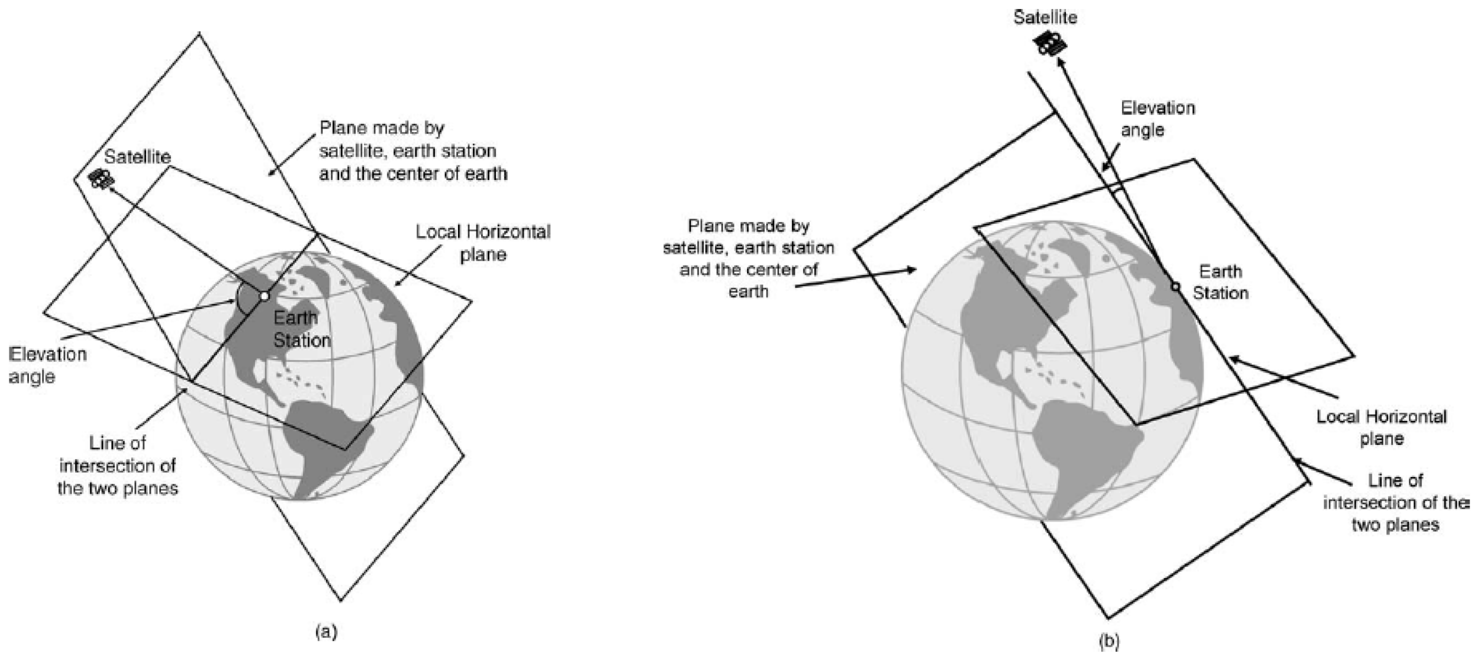


Figure 3.40 Earth station elevation angle

Elevation Angle

- The elevation angle can be computed from

$$E = \tan^{-1}\left[\frac{r - R\ \cos\theta_{\mathrm{l}}\ \cos|\theta_{\mathrm{s}} - \theta_{\mathrm{L}}|}{R\sin\{\cos^{-1}(\cos\theta_{\mathrm{l}}\cos|\theta_{\mathrm{s}} - \theta_{\mathrm{L}}|)\}}\right] - \cos^{-1}(\cos\theta_{\mathrm{l}}\cos|\theta_{\mathrm{s}} - \theta_{\mathrm{L}}|)$$

where

r = orbital radius, R = Earth's radius

θ_s = Satellite longitude, θ_L = Earth station longitude, θ_l = Earth station latitude

Computing the Slant Range

- Slant range of a satellite is defined as the range or the distance of the satellite from the Earth station.
- The smaller the elevation angle *E* of the Earth station, the larger is the slant range and the coverage angle.
  - A zero angle of elevation leads to the maximum coverage angle.
- A larger slant range means a longer propagation delay time and a greater impairment of signal quality, as the signal has to travel a greater distance through the Earth's atmosphere.

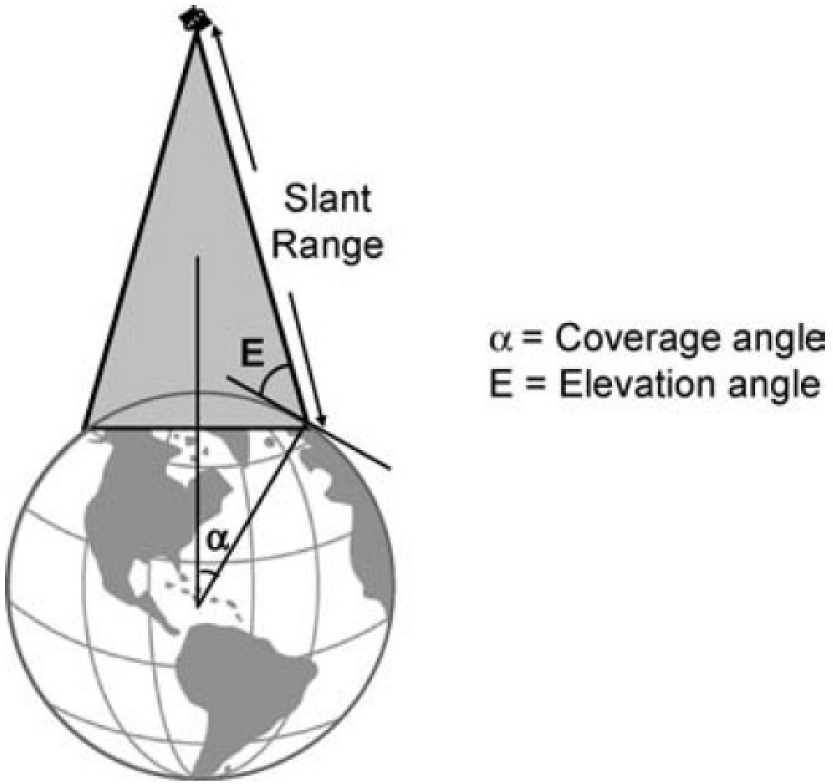


Figure 3.41 Elevation angle, slant range and coverage angle

Computing the Slant Range

- The slant range can be computed from

$$\text{Slant range } D = \sqrt{R^2 + (R+H)^2 - 2R(R+H)\ \sin\left[E + \sin^{-1}\left\{\left(\frac{R}{R+H}\right)\cos E\right\}\right]}$$

$$\text{Coverage angle } \alpha = \sin^{-1}\left\{\left(\frac{R}{R+H}\right)\cos E\right\}$$

Where

R = radius of the Earth
E = angle of elevation
H = height of the satellite above the surface of the Earth

Computing the Line-of-Sight Distance between Two Satellites

- The line-of-sight distance between two satellites placed in the same circular orbit can be computed from triangle ABC formed by the points of location of two satellites and the centre of the Earth.
- The line-of-sight distance AB in this case is given by

$$AB = \sqrt{(AC^2 + BC^2 - 2\ AC\ BC\ \cos\theta)}$$

- Angle θ will be the angular separation of the longitudes of the two satellites

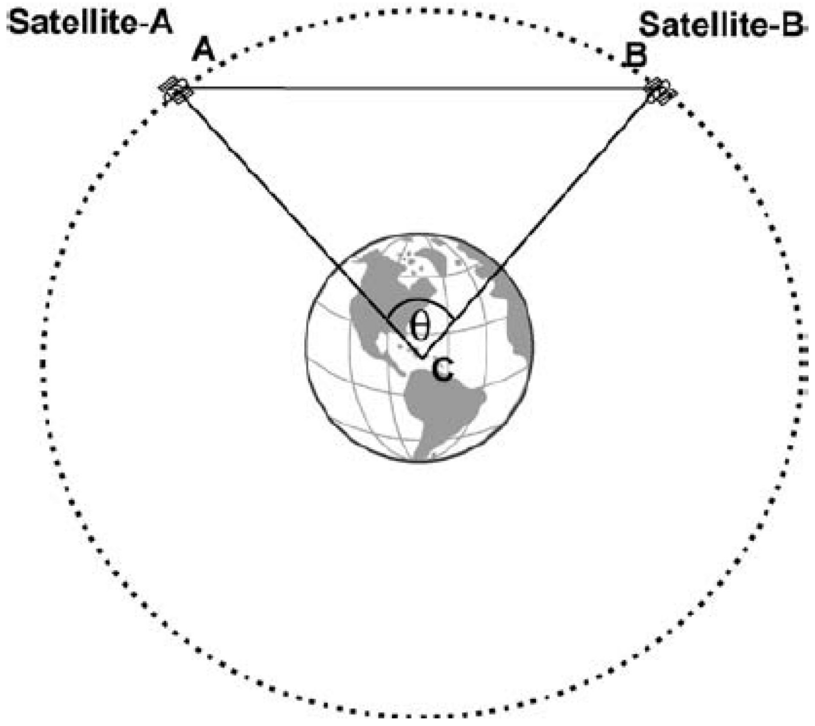


Figure 3.42 Line-of-sight distance between two satellites

Computing the Line-of-Sight Distance between Two Satellites

- The maximum line-of-sight distance between these two satellites occurs when the satellites are placed so that the line joining the two becomes tangent to the Earth's surface.
- In this the case, the maximum line-of-sight distance (AB) equals OA + OB, which further equals 2OA or 2OB as OA=OB.

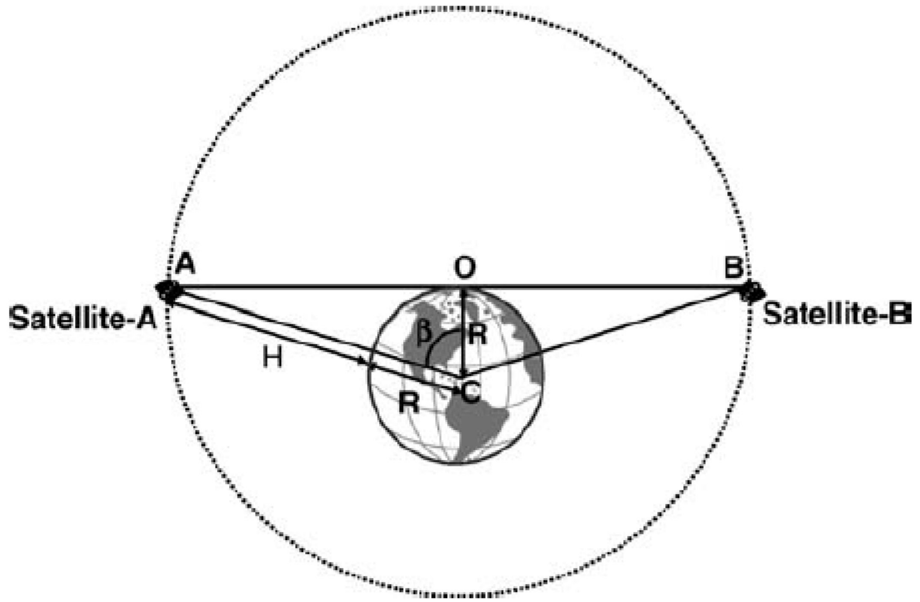


Figure 3.43 Maximum line-of-sight distance between two satellites

Computing the Line-of-Sight Distance between Two Satellites

- If R is the radius of the Earth and H is the height of satellites above the surface of the Earth, then

$$\mathrm{OA} = \mathrm{AC}\ \sin\beta = (R+H)\ \sin\beta$$

Now

$$\beta = \cos^{-1}\left(\frac{R}{R+H}\right)$$

Therefore

$$\mathrm{OA} = (R+H)\ \sin\left[\cos^{-1}\left(\frac{R}{R+H}\right)\right]$$

and

$$\text{Maximum line-of-sight distance} = 2(R+H)\ \sin\left[\cos^{-1}\left(\frac{R}{R+H}\right)\right]$$

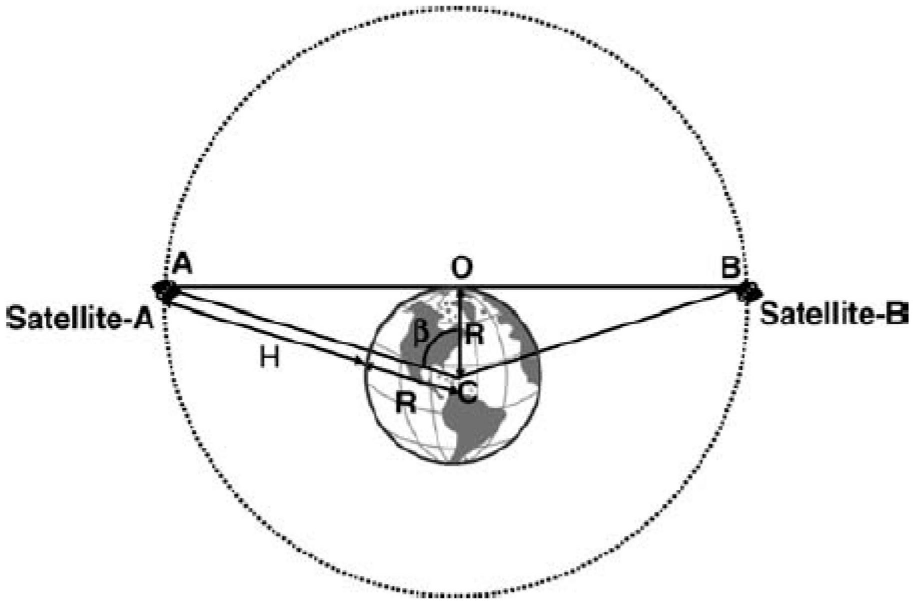


Figure 3.43 Maximum line-of-sight distance between two satellites

Problem 3.7

- Determine the maximum possible line-of-sight distance between two geostationary satellites orbiting the Earth at a height of 36000 km above the surface of the Earth. Assume the radius of the Earth to be 6370 km.

Solution:

Maximum line-of-sight distance can be computed from

$$2(R+H)\sin\left[\cos^{-1}\left\{\frac{R}{(R+H)}\right\}\right]$$

where

R = Radius of Earth
H = Height of satellite above surface of Earth

Therefore, Maximum line-of-sight distance is given by:

$$2(6370+36\,000)\sin[\cos^{-1}\{6370/(6370+36\,000)\}]$$

$$= 2(42\,370)\sin[\cos^{-1}\{6370/42\,370\}] = 84\,740 \times 0.989 = 83\,807.86\text{ km}$$

Reference

- Anil K. Maini, Varsha Agrawal, *Satellite Communications*, Wiley India Pvt. Ltd., 2015, ISBN: 978-81-265-2071-8.