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Bhatkal, Karnataka, India

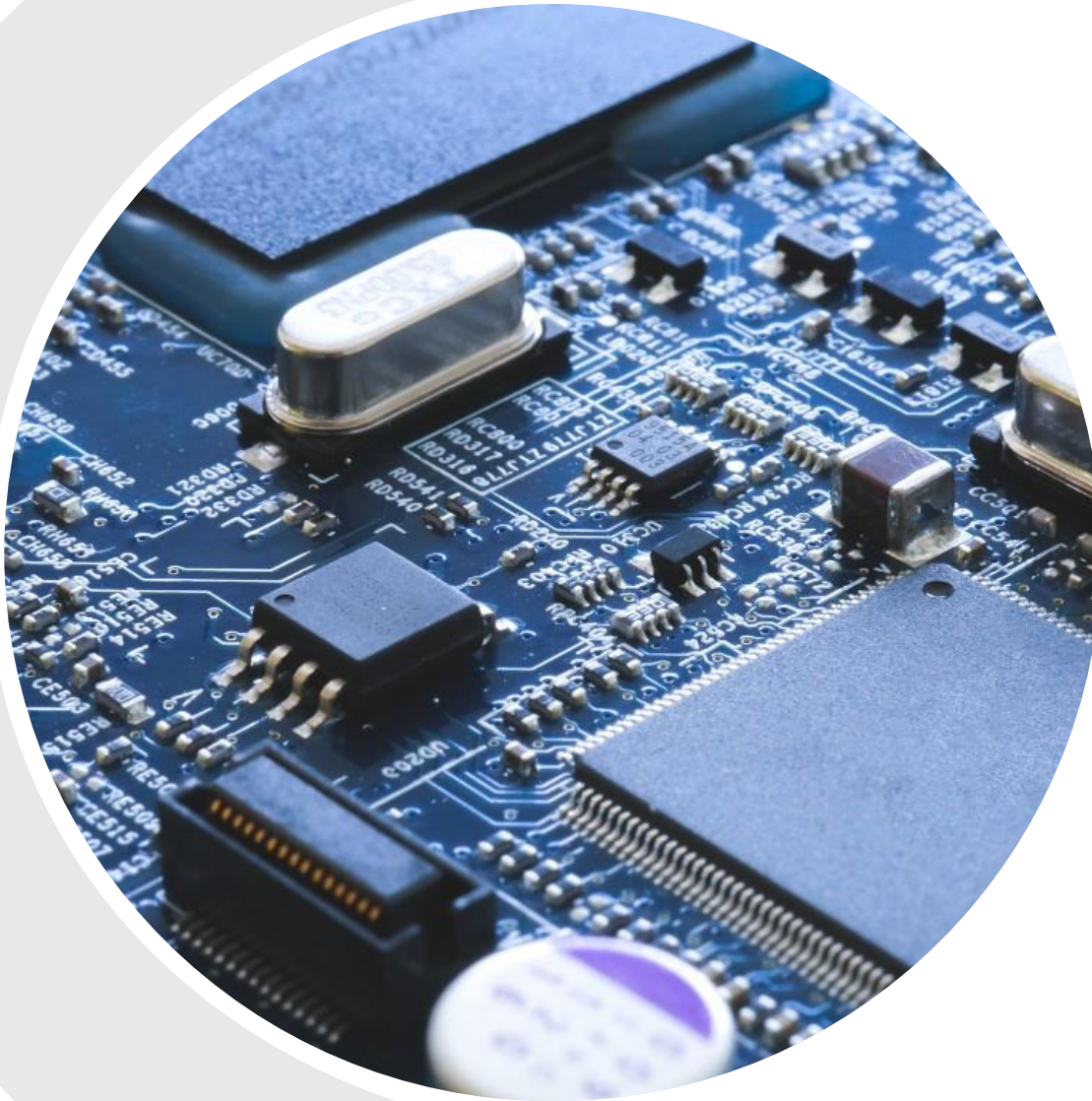
BASIC ELECTRONICS

(BBEE103/BBEE203)

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MODULE – 1

Semiconductor Diodes and Applications

Syllabus

Semiconductor Diodes: Introduction, PN Junction diode, Characteristics and Parameters, Diode Approximations, DC Load Line Analysis

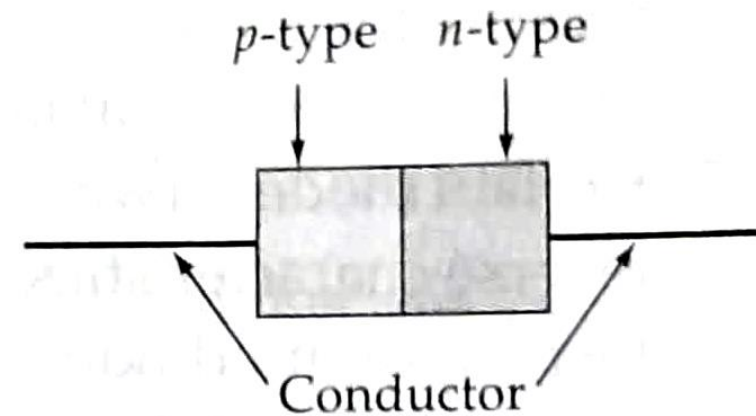
Diode Applications: Introduction, Half Wave Rectification, Full Wave Rectification, Full Wave Rectifier Power Supply: Capacitor Filter Circuit, RC- π Filter (includes numerical)

Zener Diodes: Junction Breakdown, Circuit Symbol and Package, Characteristics and Parameters, Equivalent Circuit, Zener Diode Voltage Regulator.

Semiconductor Diodes

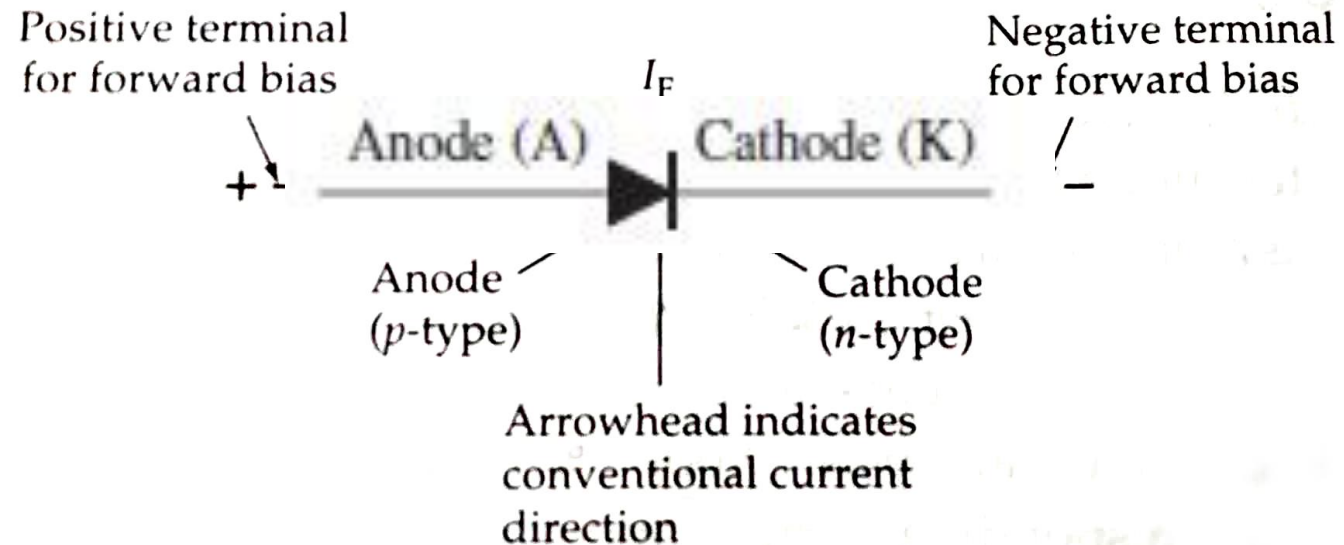
PN Junction Diode

- A PN junction diode is a two terminal unidirectional device with a p -type anode and an n -type cathode.
- A PN junction provided with copper wire (conductor) connecting leads becomes an electronic device known as a *diode*.
- Diodes are used in a wide range of applications like rectification, voltage regulation, protection against high voltage and wave shaping.



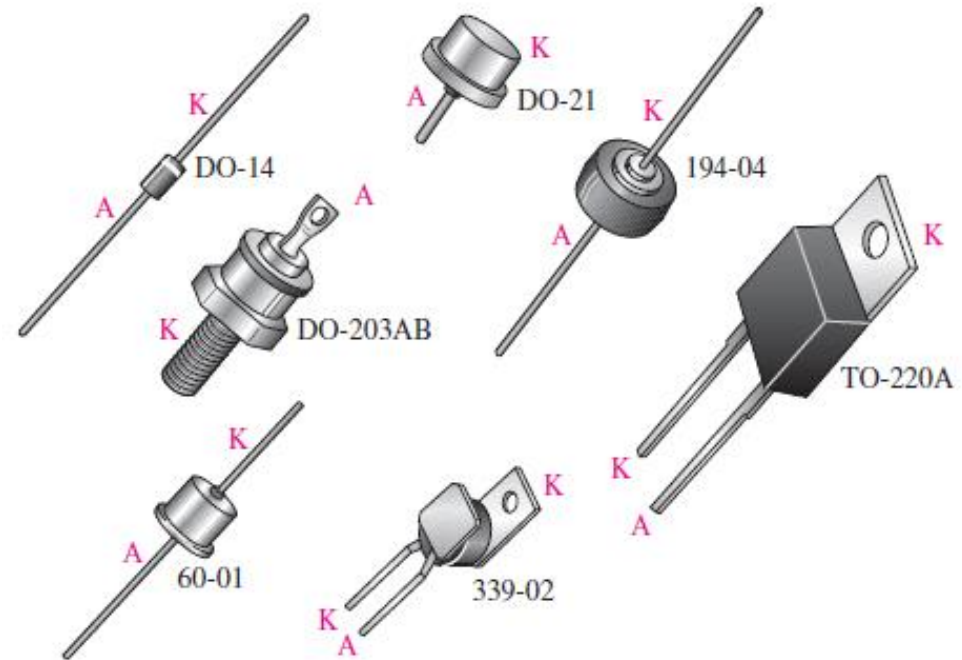
A semiconductor diode is a PN junction with conductors on each side of the junction for connecting the device to a circuit

PN Junction Diode



Symbol of a diode

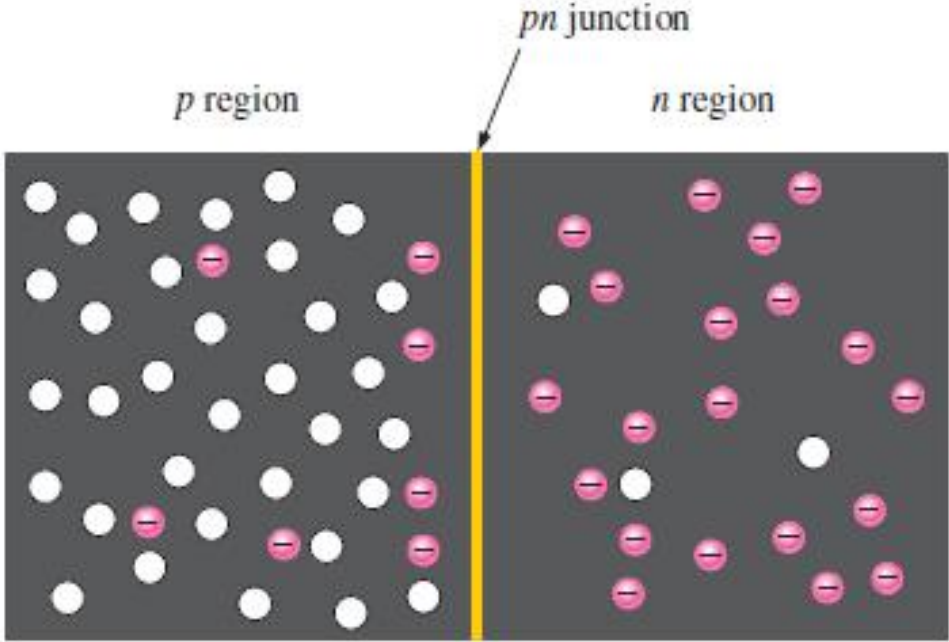
PN Junction Diode



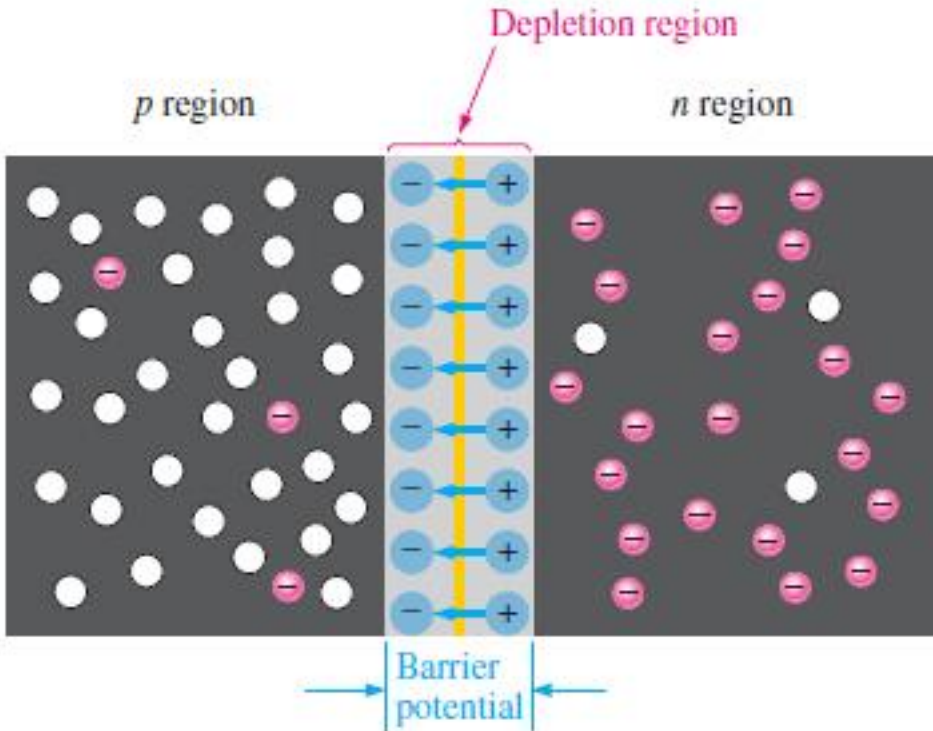
Typical diode packages

Diode Operation and Characteristics

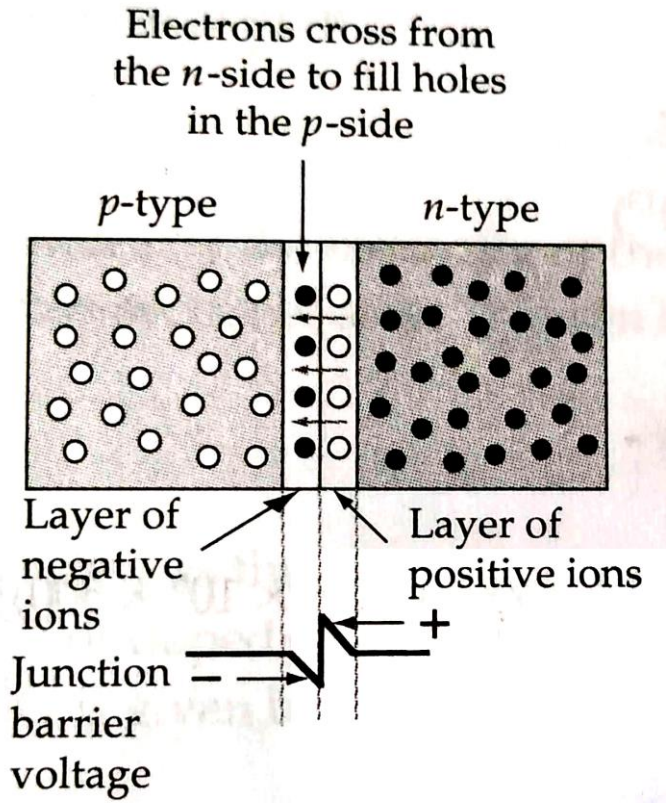
Unbiased PN Junction



Unbiased PN Junction



Unbiased PN Junction



Unbiased PN Junction

- Since holes and electrons are close together at the junction, some free electrons from the n -side are attracted across the junction to fill adjacent holes on the p -side.
 - They are said to *diffuse* across the junction from a region of high carrier concentration to one of low concentration.
- The free electrons crossing the junction create negative ions on the p -side by giving some atoms one more electron than their total number of protons.
- The electrons also leave positive ions (atoms with one lesser electron than the number of protons) behind them on the n -side.

Unbiased PN Junction

Depletion Region

- The movement of charge carriers across the junction leaves a layer on each side that is depleted of charge carriers.
 - This region is called the *depletion region*.
- On the n-side, the depletion region is made up of donor impurity atoms that have become positively charged by losing the free electron associated with them.
- On the p-side, the depletion region is made up of acceptor impurity atoms that have become negatively charged by losing the hole associated with them.

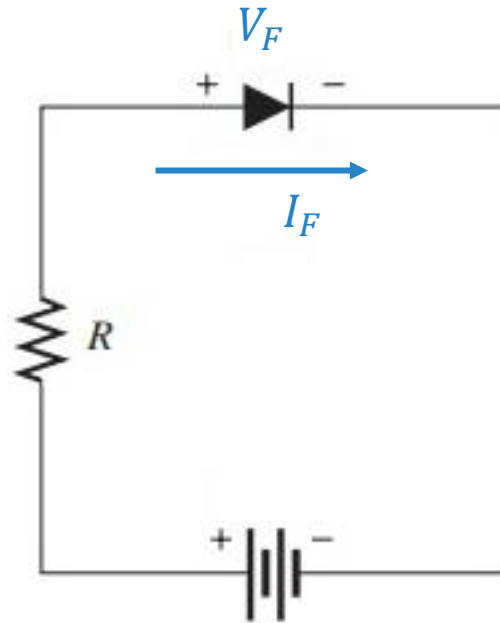
Unbiased PN Junction

Barrier Voltage

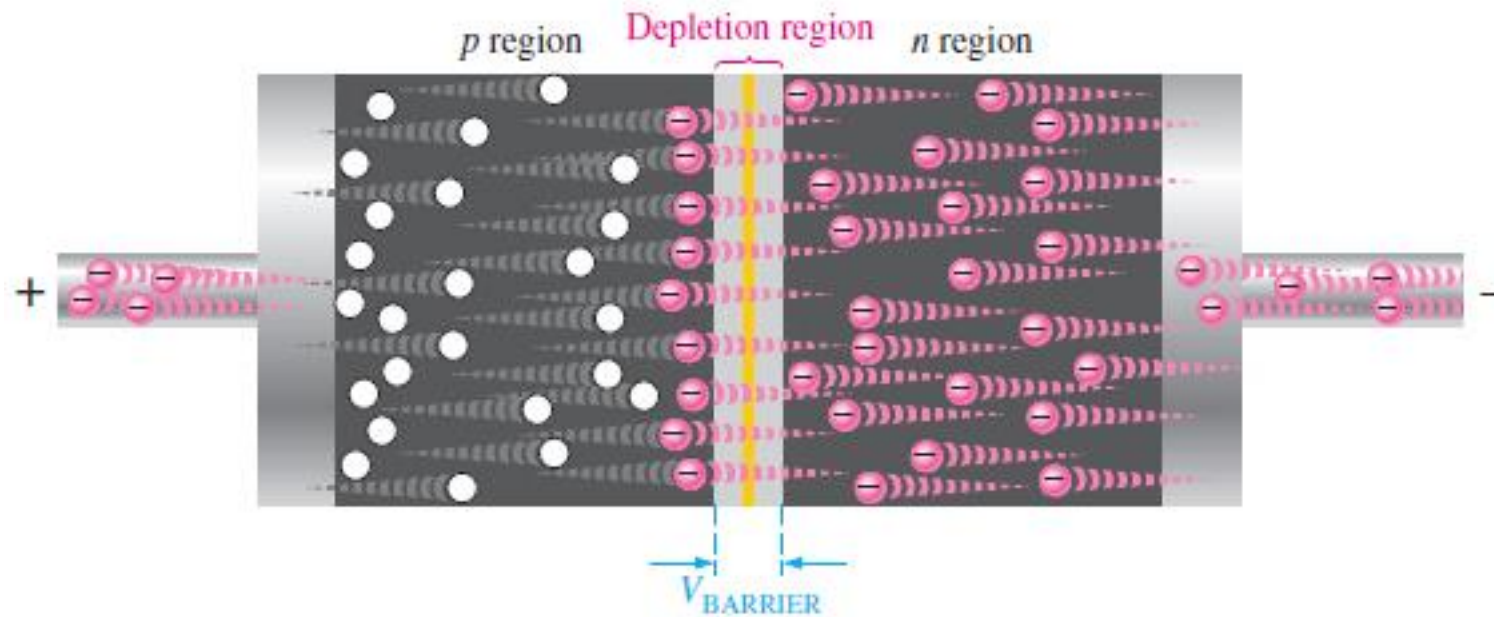
- The n-type and p-type materials are both electrically neutral before the diffusion of charge carriers across the junction.
- The diffusion of charge carriers creates positive and negative ions on n -side and p -side, which results in an electric field near the junction.
- This creates a potential difference across the depletion region which acts as a barrier.
 - This region is called the *barrier voltage* or *barrier potential* or *junction potential*, which is negative on the p -side and positive on the n -side.
- Typical barrier voltages at 25°C are 0.3 V for germanium and 0.7 V for silicon junctions.

Forward Biasing of Diode

- A diode is said to be forward biased when the positive terminal of the battery is connected to the p -side and negative terminal to the n -side.

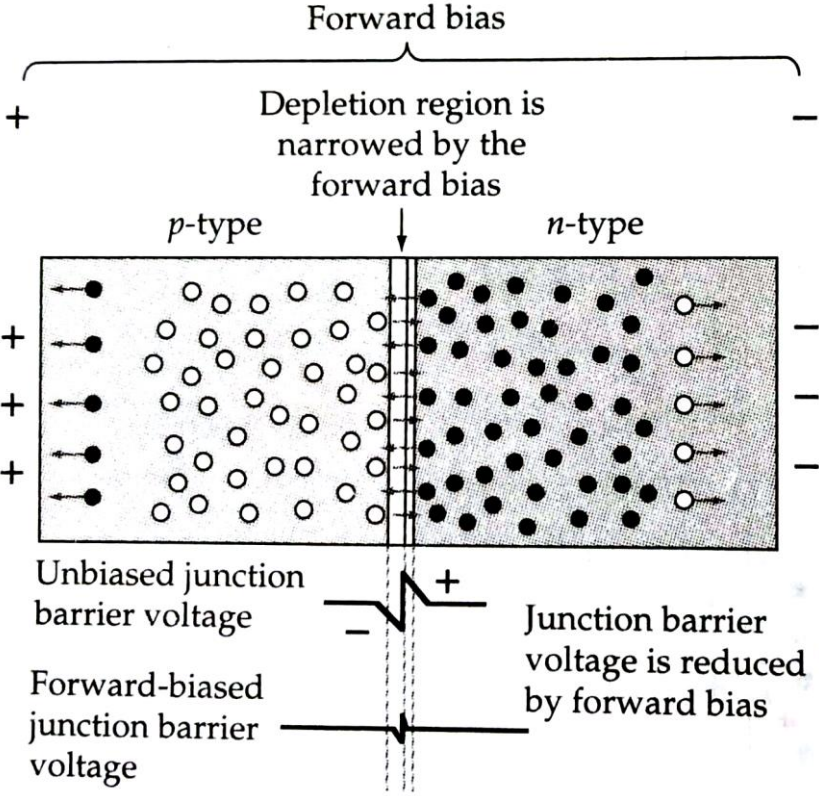


Forward Biasing of Diode



A forward-biased diode showing the flow of majority carriers and the voltage due to the barrier potential across the depletion region.

Forward Biasing of Diode



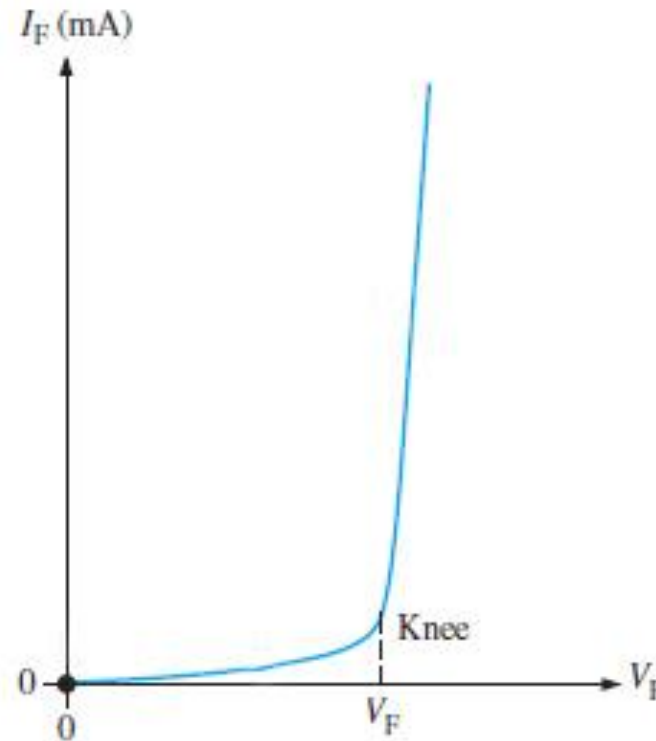
Forward Biasing of Diode

- The holes on the p -side, being positively charged particles, are repelled from the positive terminal and driven toward the junction.
- Similarly, the electrons on the n -side are repelled from the negative terminal toward the junction.
- As a result, the width of the depletion region and the barrier voltage are both reduced.
- When the applied bias voltage is progressively increased from zero, the barrier voltage gets smaller until it effectively disappears and charge carriers easily flow across the junction.

Forward Biasing of Diode

- Electrons from the n-side are now attracted to the positive bias terminal on the p-side, and holes from the p-side are attracted to the negative terminal on the n-side.
- The majority charge carrier current flows, and the junction is said to be *forward biased*.

Forward Characteristic



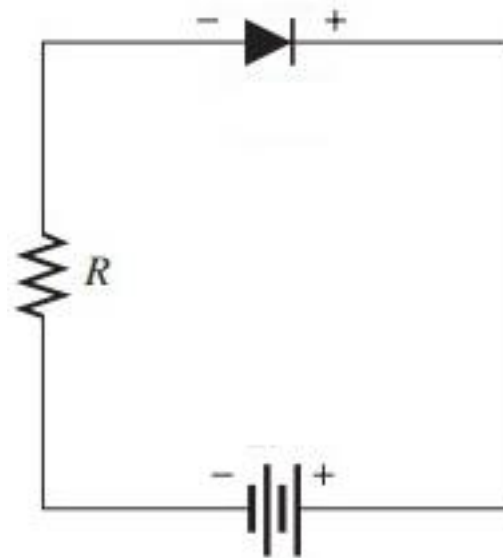
Forward Characteristic (V-I characteristic of a forward biased diode)

Forward Characteristic

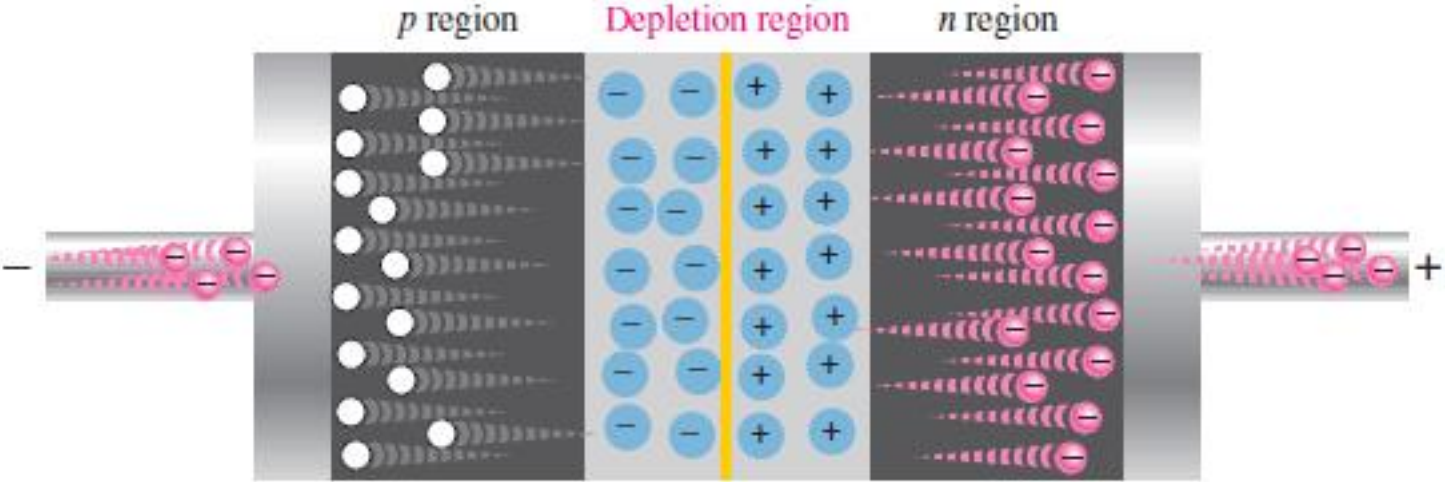
- From the forward characteristic, it can be seen that there is a very little forward current until V_F exceeds the junction barrier voltage (0.3 V for germanium and 0.7 V for silicon).
- When V_F is increased from zero toward the knee of the characteristic, the barrier voltage is progressively overcome, allowing more majority charge carriers to flow across the junction.
- Above the knee of the characteristic, I_F increases almost linearly with increase in V_F .

Reverse Biasing of Diode

- A diode is said to be reverse biased when the positive terminal of the battery is connected to the n -side and negative terminal to the p -side.

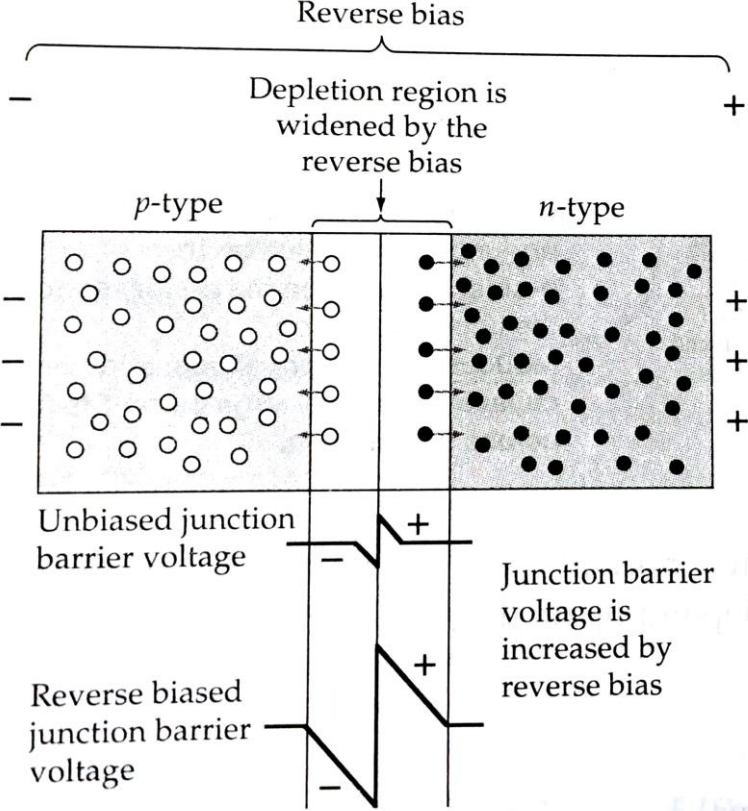


Reverse Biasing of Diode



A reverse-biased diode

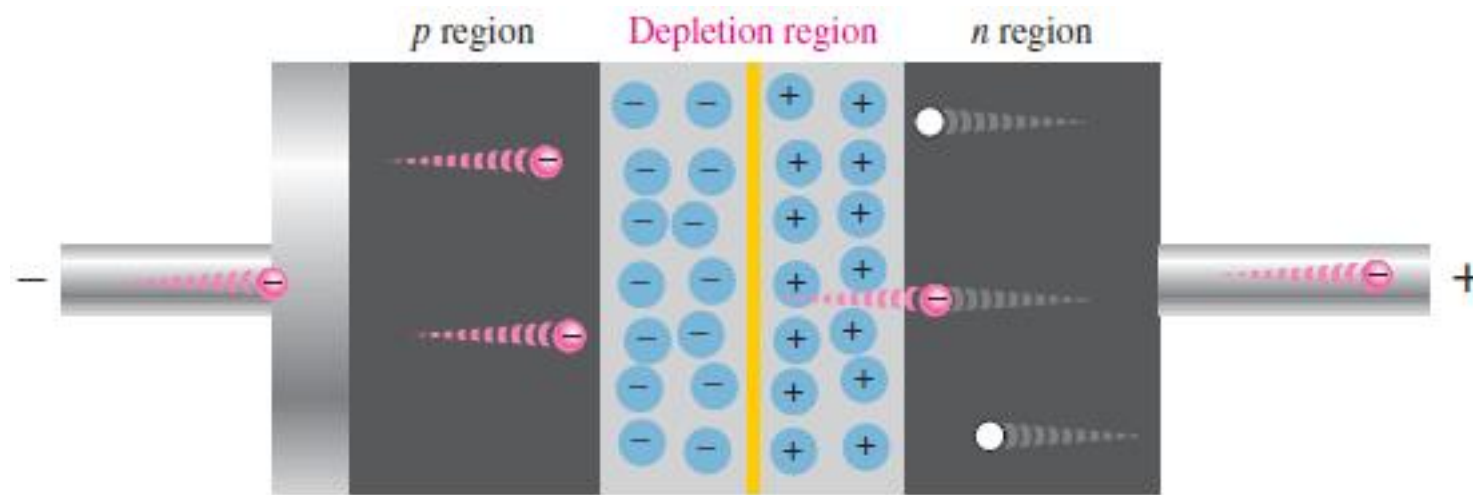
Reverse Biasing of Diode



Reverse Biasing of Diode

- Electrons from the n -side are attracted to the positive terminal away from the junction, and holes on the p -side are attracted to the negative terminal away from the junction.
- This causes the depletion region to be widened and the barrier voltage to be increased.
- With the barrier voltage increase, there is no possibility of a majority charge carrier current flow across the junction, and the junction is said to be *reverse biased*.
- Although, there is very small reverse current and hence the reverse biased PN junction can be said to have a high resistance.

Reverse Biasing of Diode



The extremely small reverse current in a reverse-biased diode is due to the minority carriers from thermally generated electron-hole pairs.

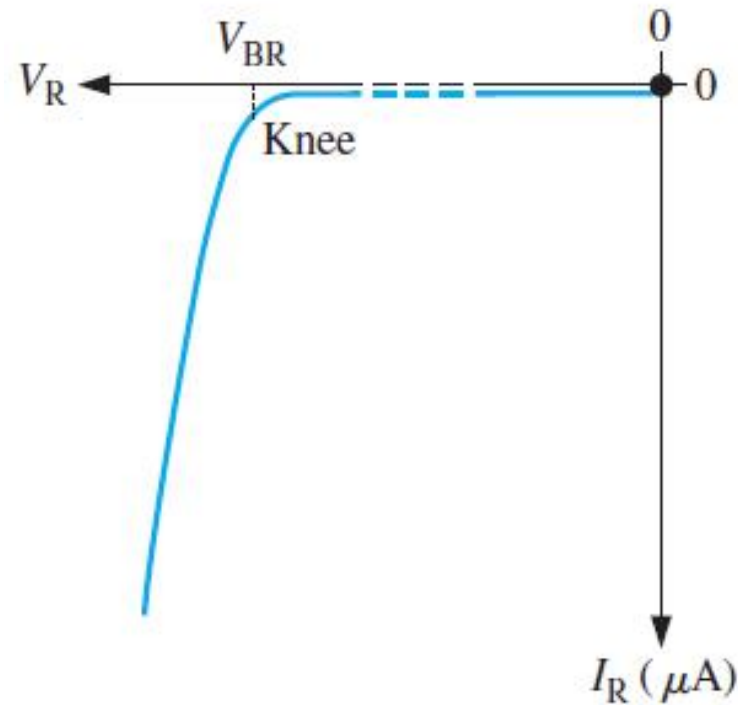
Reverse Biasing of Diode

- Although there is no possibility that a majority charge carrier current can flow across a reverse biased junction, minority carriers generated on each side can still cross the junction.
- Electrons in the p -side are attracted across the junction to the positive voltage on the n -side, and holes on the n -side may flow across to the negative voltage on the p -side.
- Since only a very small reverse bias voltage is necessary to direct all available minority carriers across the junction, further increase in bias voltage does not increase the current level.
 - This current is known as *reverse saturation current*.
 - The reverse saturation current is normally a very small quantity, ranging from nanoamperes (nA) to microamperes (μ A).

Reverse Biasing of Diode

- If the reverse bias voltage is increased to a value called the *breakdown voltage*, the diode breaks down and the reverse current will increase drastically.

Reverse Characteristic

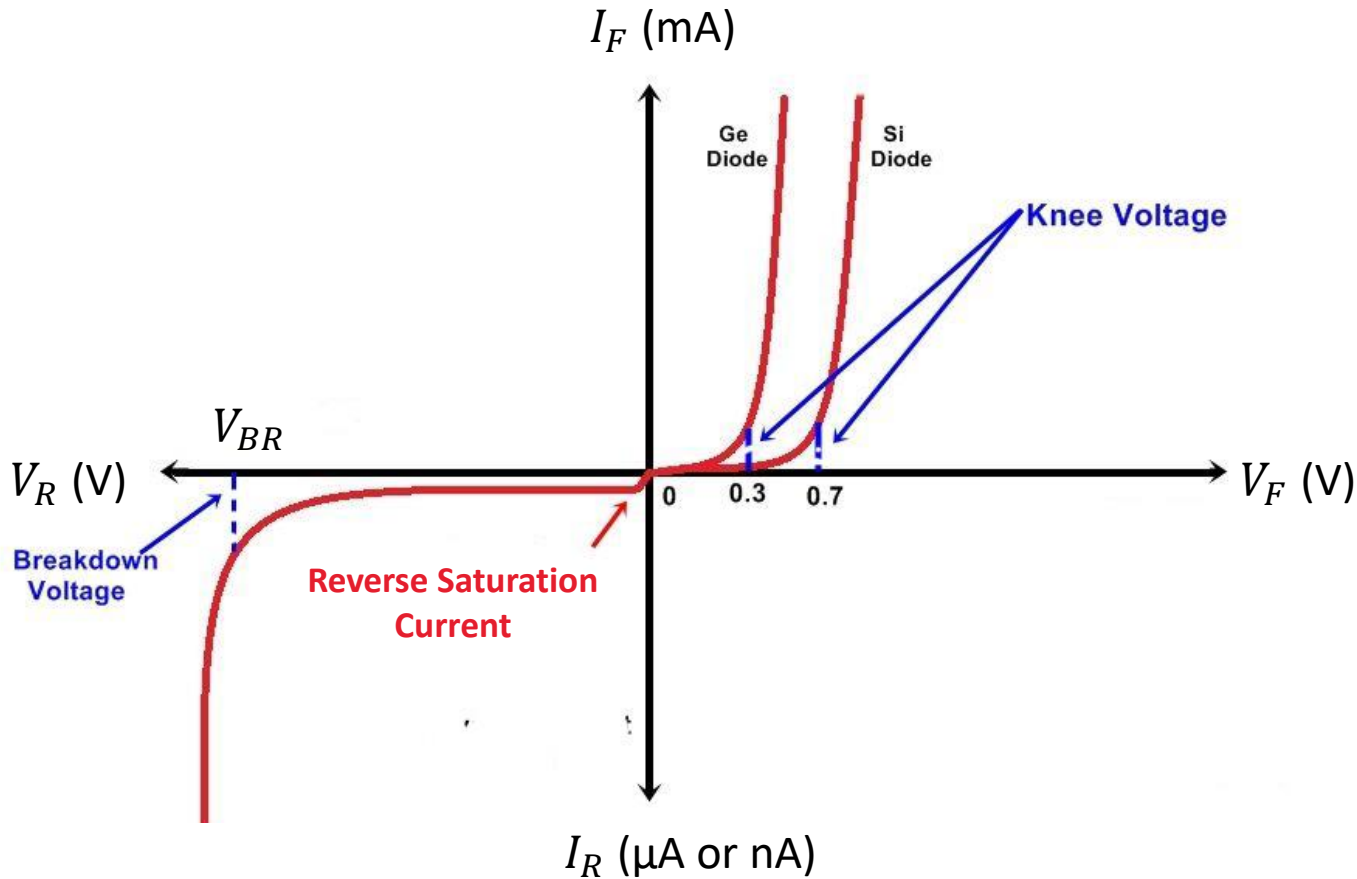


Reverse Characteristic (V-I characteristic of a reverse biased diode)

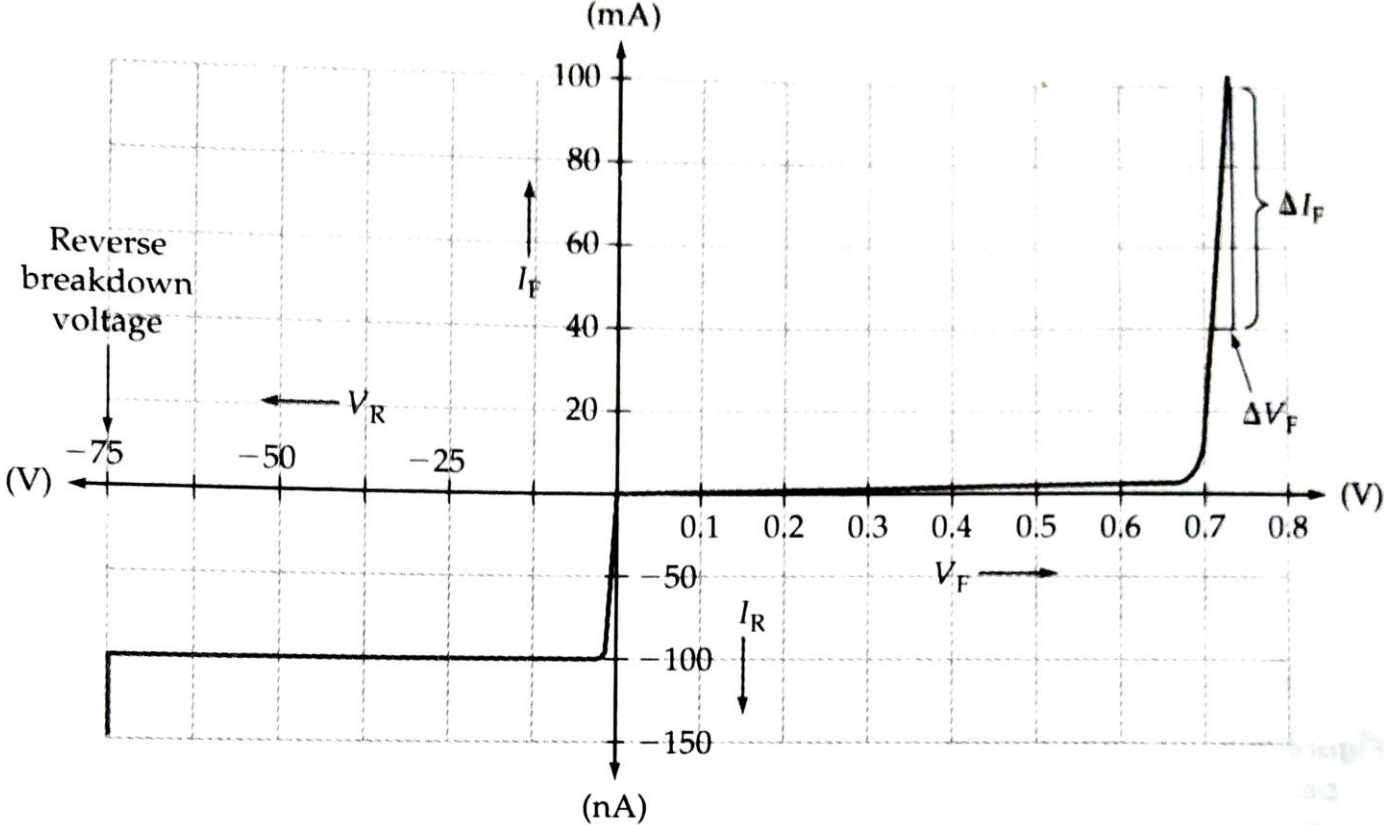
Reverse Characteristic

- From the reverse characteristic, it can be seen that I_R is largely a minority charge carrier reverse saturation current.
- A small increase in I_R can occur with increasing reverse bias voltage, as a result of minority charge carriers leaking along the junction surface.
- Normally, the reverse current is very small and it can be neglected.
- However, if the reverse bias voltage is increased to a value called the *breakdown voltage*, the reverse current will drastically increase.

Forward and Reverse Characteristic

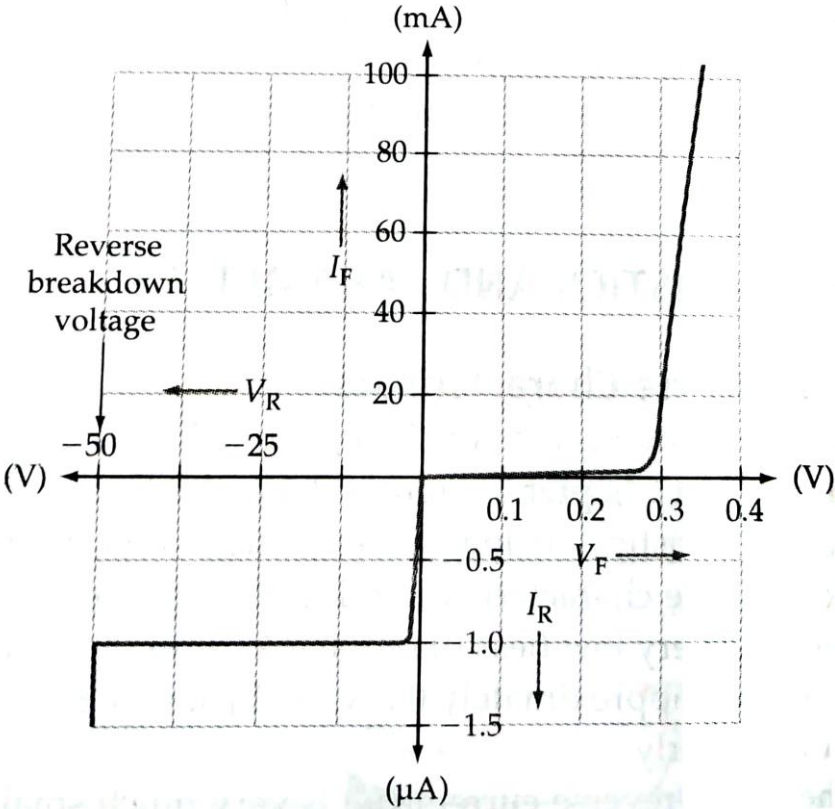


Forward and Reverse Characteristic



V-I characteristic of a silicon diode

Forward and Reverse Characteristic



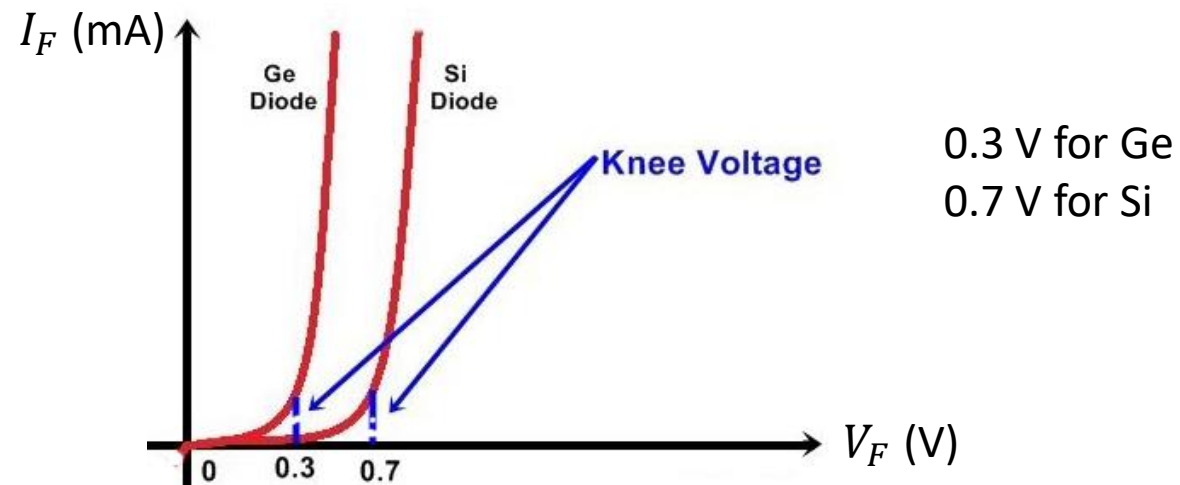
V-I characteristic of a germanium diode

Diode Parameters

Diode Parameters

Knee Voltage

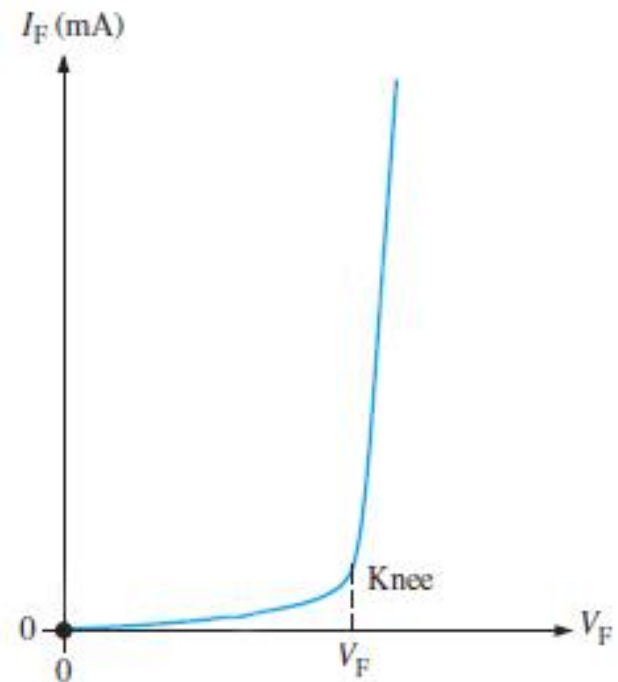
- It is the small forward voltage applied to a forward biased diode at which current starts increasing exponentially.
 - Also called *threshold voltage* (V_T) or *cut-in voltage*.



Diode Parameters

Forward Voltage Drop (V_F)

- It is the voltage drop across a forward biased diode.



0.3 V for Ge
0.7 V for Si

Diode Parameters

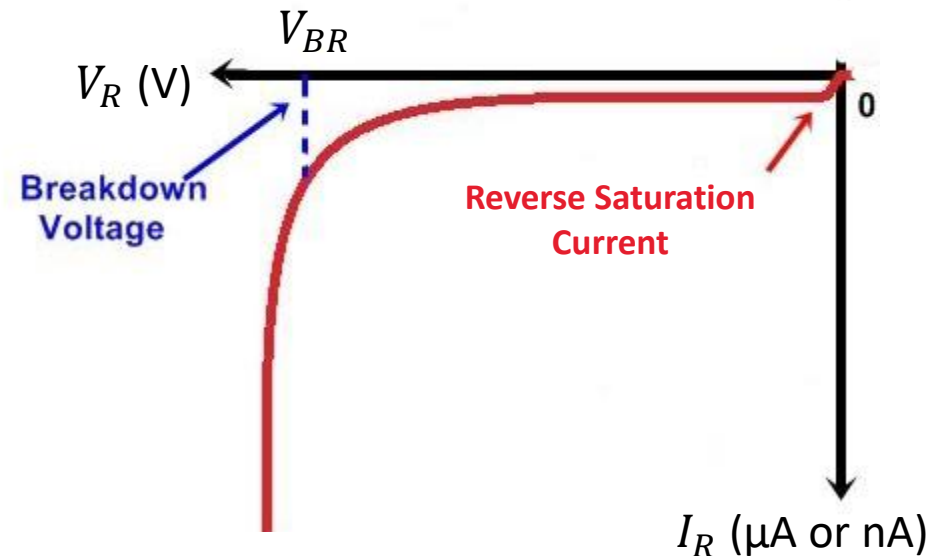
Maximum Forward Current ($I_{F(max)}$)

- It is the maximum current that a forward biased diode can conduct without burning out.

Diode Parameters

Reverse Saturation Current ($I_{R(sat)}$ or I_S)

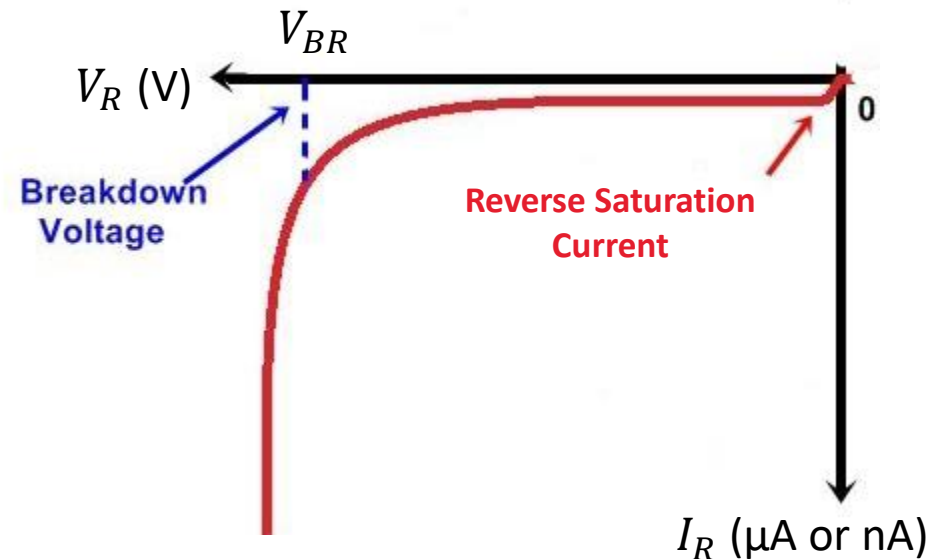
- It is the small amount of constant reverse current flowing through the diode when it is reverse biased.



Diode Parameters

Reverse Breakdown Voltage (V_{BR})

- It is the value of reverse voltage at which the diode breaks down and the reverse current increases drastically.
 - The diode gets damaged due to breakdown.



Diode Parameters

Peak Inverse Voltage (PIV)

- It is the maximum value of reverse voltage that can be applied to the diode without causing breakdown.
 - It is also called PIV rating of the diode.

Diode Parameters

Maximum Power Rating (MPR)

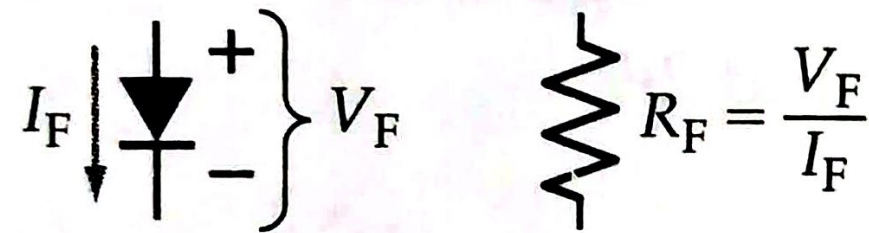
- It is the maximum power that the diode can dissipate safely, without increasing the junction temperature above its limiting value.

Static Resistance

- Static resistance is also called dc resistance.

Static Forward Resistance (R_F)

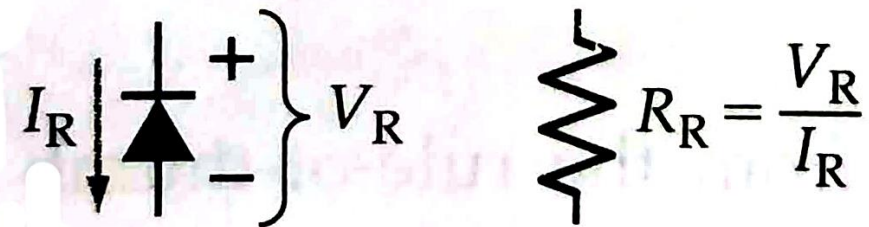
- It is the resistance offered by the forward biased PN junction diode and calculated at a particular point on the forward characteristics.



(a) Forward resistance

Static Reverse Resistance (R_R)

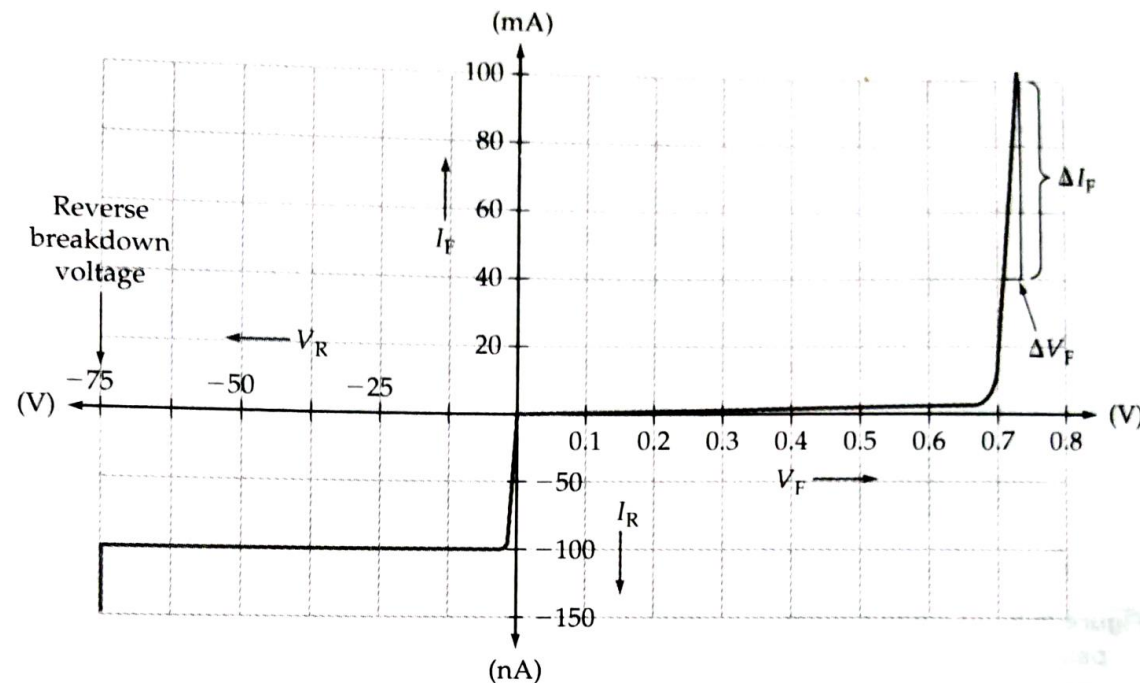
- It is the resistance offered by the reverse biased PN junction diode and calculated at a particular point on the reverse characteristics.



(b) Reverse resistance

Static Resistance – Numerical Example

Calculate the forward and reverse resistances offered by a silicon diode with the characteristics as shown in the figure at $I_F = 100 \text{ mA}$ and at $V_R = 50 \text{ V}$.



Static Resistance – Numerical Example

Solution:

From the characteristics, at $I_F = 100 \text{ mA}$, $V_F \approx 0.75 \text{ V}$

The forward resistance,

$$R_F = \frac{V_F}{I_F}$$

$$R_F = \frac{0.75 \text{ V}}{100 \text{ mA}}$$

$$R_F = 7.5 \Omega$$

Static Resistance – Numerical Example

From the characteristics, at $V_R = 50\text{ V}$, $I_R \approx 100\text{ nA}$

The reverse resistance,

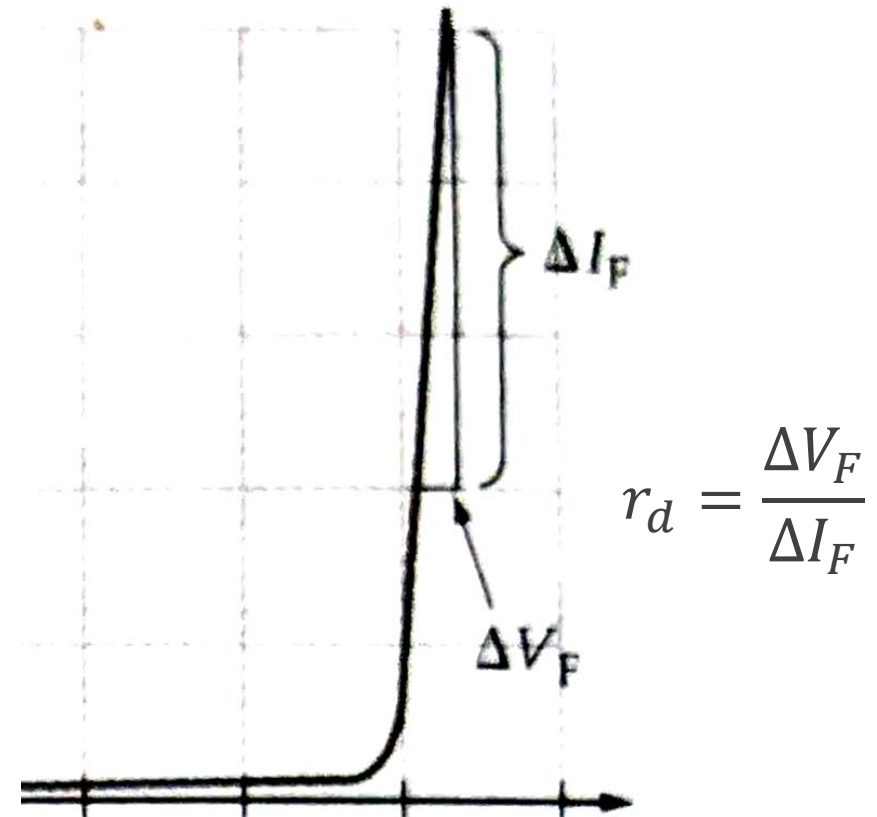
$$R_R = \frac{V_R}{I_R}$$

$$R_R = \frac{50\text{ V}}{100\text{ nA}}$$

$$R_R = 500\text{ M}\Omega$$

Dynamic Resistance

- The dynamic resistance of the diode is the resistance offered to the changing levels of forward voltage.
 - It is indicated by r_d .
- It is the reciprocal of the slope of the forward characteristics beyond the knee.
- Dynamic resistance is also called *incremental resistance* or *ac resistance*.



Dynamic Resistance

- The dynamic resistance can also be calculated from the rule-of-thumb equation

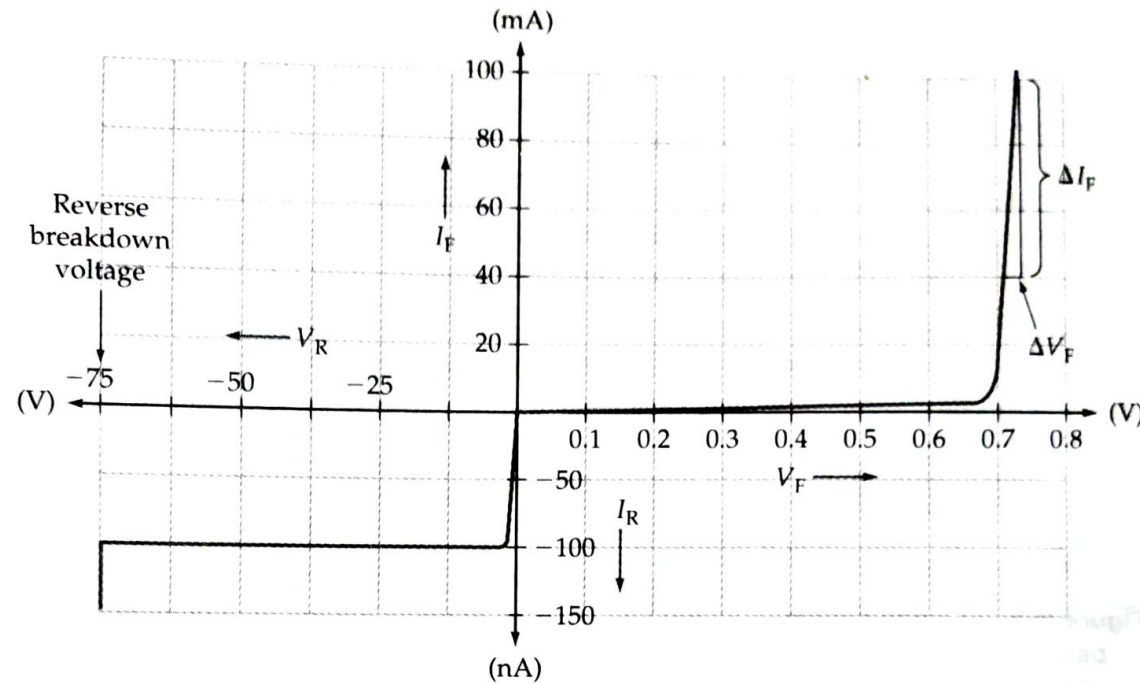
$$r'_d = \frac{26 \text{ mV}}{I_F}$$

- For example, the dynamic resistance for a diode passing a 1 mA forward current is

$$r'_d = \frac{26 \text{ mV}}{I_F} = \frac{26 \text{ mV}}{1 \text{ mA}} = 26 \Omega$$

Dynamic Resistance – Numerical Example

Determine the dynamic resistance at a forward current of 70 mA for the diode characteristics given in the figure.



Dynamic Resistance – Numerical Example

Solution:

From the characteristics, at $I_F = 70 \text{ mA}$,

$$\Delta I_F = 60 \text{ mA} \text{ and } \Delta V_F \approx 0.025 \text{ V}$$

The dynamic resistance,

$$r_d = \frac{\Delta V_F}{\Delta I_F}$$

$$r_d = \frac{0.025 \text{ V}}{60 \text{ mA}}$$

$$r_d = 0.416 \Omega$$

Dynamic Resistance – Numerical Example

The dynamic resistance can also be calculated as,

$$r'_d = \frac{26 \text{ mV}}{I_F}$$

$$r'_d = \frac{26 \text{ mV}}{70 \text{ mA}}$$

$$r'_d = 0.371 \Omega$$

Diode Approximations

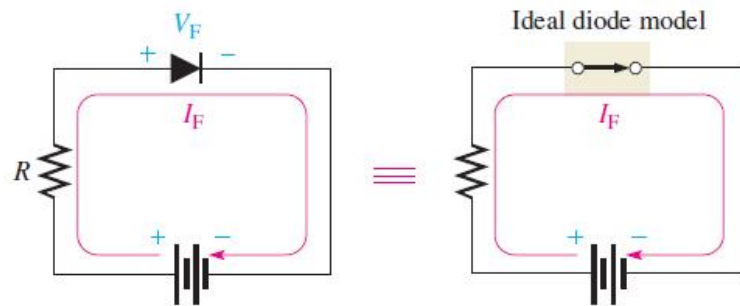
Equivalent Circuits of Diode

- An *equivalent circuit* for a device is a circuit that represents the device behaviour.
- A *diode equivalent circuit* may be substituted for the device when investigating a circuit containing the diode.

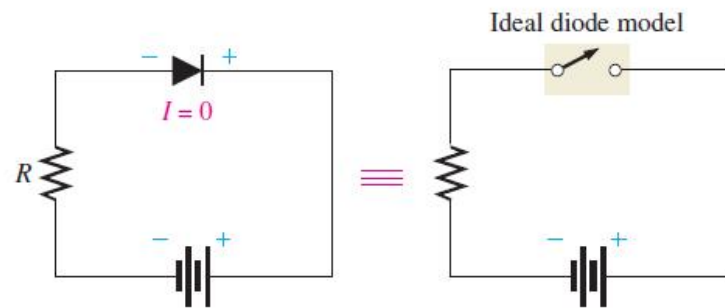
Ideal Model

- An *ideal diode* (or perfect diode) would have zero forward resistance and zero forward voltage drop.
- It would also have an infinitely high reverse resistance, which would result in zero reverse current.
- The ideal model of a diode is the least accurate approximation and can be represented by a simple switch.
 - When the diode is forward biased, it ideally acts like a closed switch (ON).
 - When the diode is reverse biased, it ideally acts like an open switch (OFF).
- The barrier voltage, the forward dynamic resistance and the reverse current are all neglected.

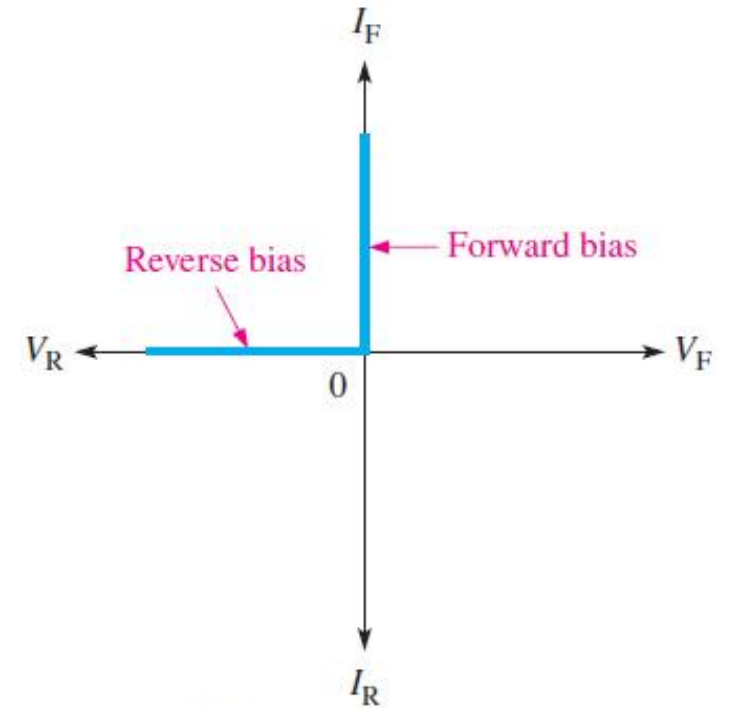
Ideal Model



(a) Forward bias



(b) Reverse bias



(c) Ideal V - I characteristic curve (blue)

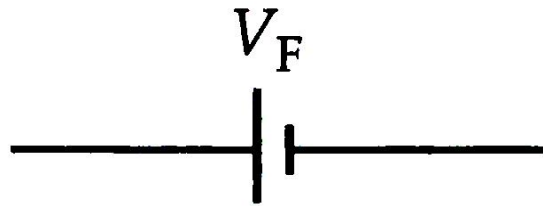
Ideal Model of a Diode

An ideal diode has $V_F = 0$, $I_R = 0$ and $r_d = 0$

Approximate Model (Practical Model)

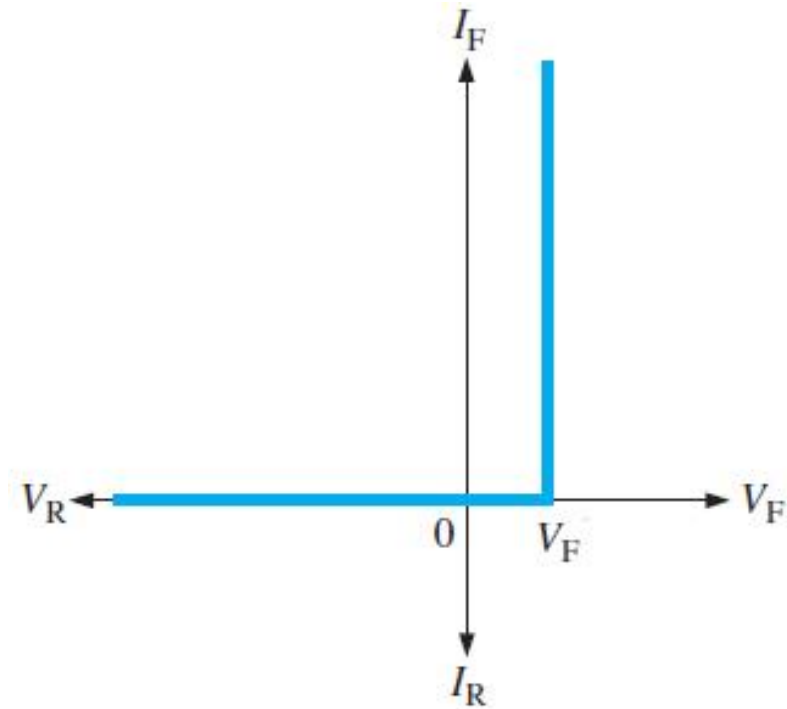
- The approximate model or practical model of diode includes the barrier voltage.
- A forward biased diode is assumed to have a constant forward voltage drop (V_F) and negligible resistance.
- In this case, the diode equivalent circuit is assumed to be a voltage cell with a voltage V_F .
- In circuits with supply voltages much larger than the forward voltage drop, V_F can be assumed to be constant without introducing any serious errors.
- In approximate or practical model, the forward dynamic resistance and the reverse current are neglected.

Approximate Model (Practical Model)



Approximate Model of Diode

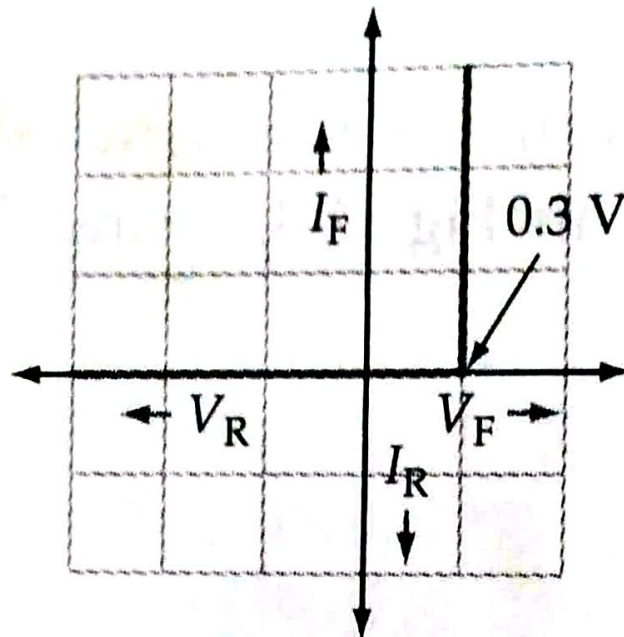
(Basic dc equivalent circuit)



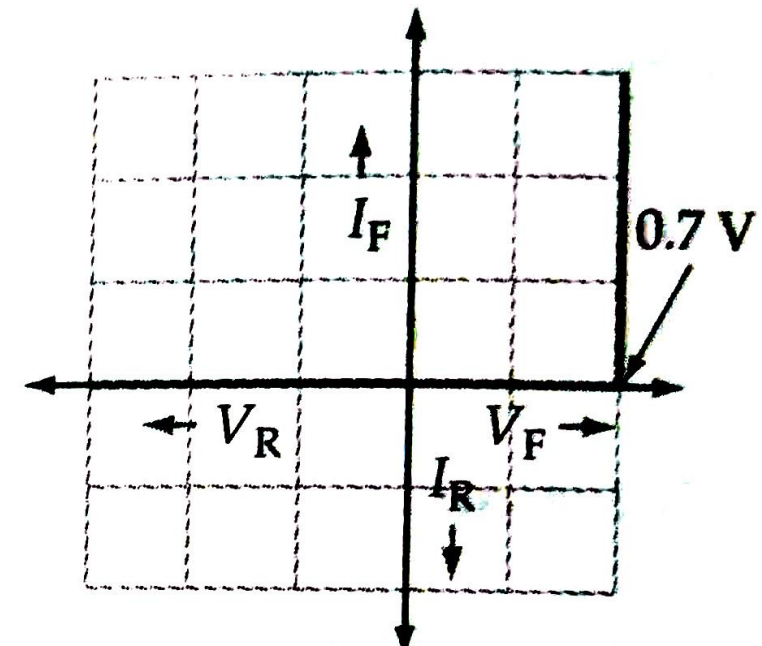
Approximate Characteristics

In practical diode, V_F is considered, $I_R = 0$ and $r_d = 0$

Approximate Model (Practical Model)



Approximate characteristics of Germanium diode

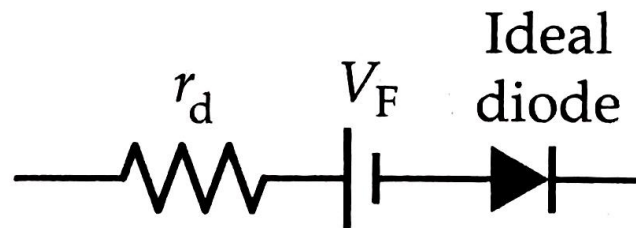


Approximate characteristics of Silicon diode

Piecewise Linear Model (Complete Model)

- The *complete model* of a diode is the most accurate approximation and includes the barrier voltage (forward voltage drop (V_F) and the small forward dynamic resistance (r_d).
- The complete dc equivalent circuit includes the diode dynamic resistance (r_d) in series with the voltage cell with a voltage V_F .
- This takes account of the small variations in V_F that occur with change in forward current.
- The *piecewise linear characteristic* is the straight-line approximation of the forward characteristic and can be employed when the forward characteristic of a diode is not available.

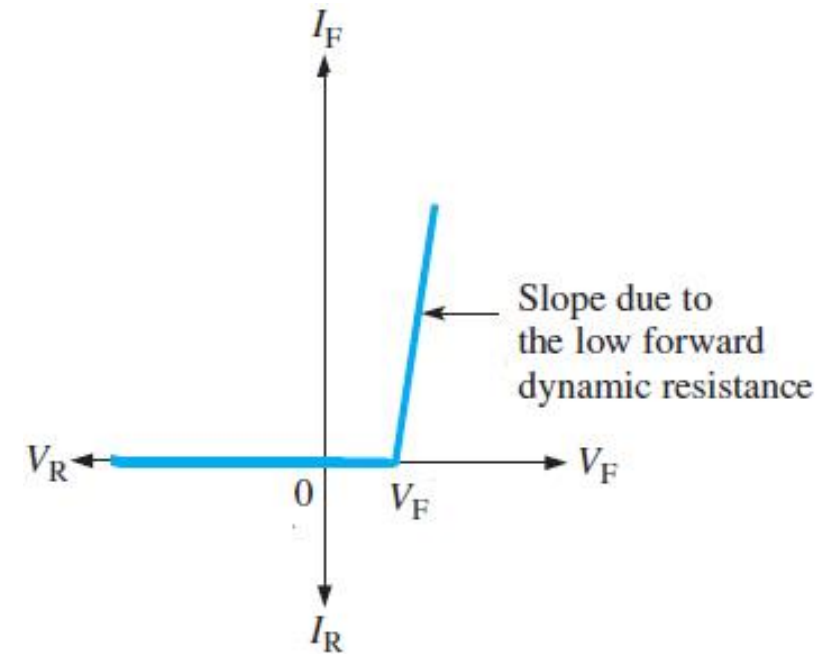
Piecewise Linear Model (Complete Model)



Complete Model of Diode

(Complete dc equivalent circuit)

Note: An ideal diode is also included to show that current flows only in one direction.

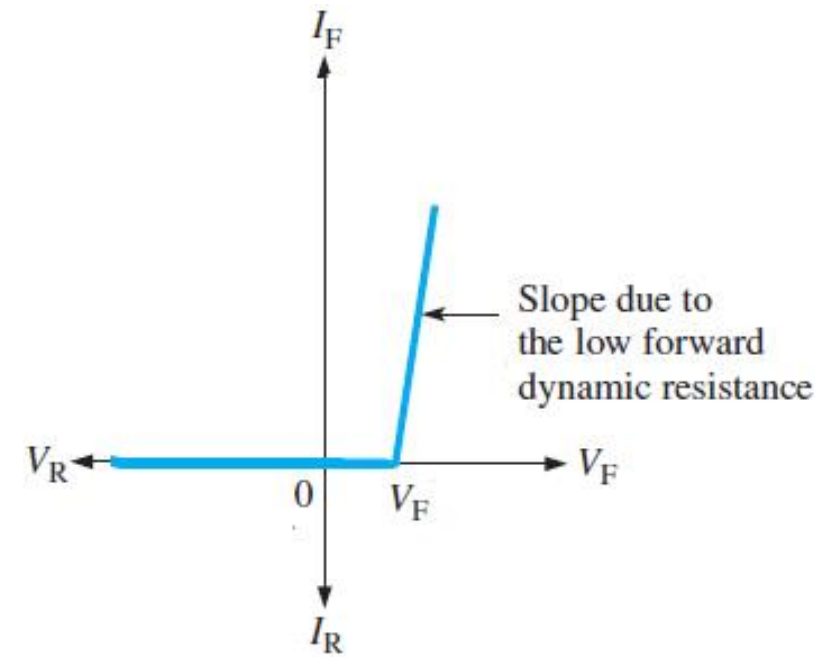


Piecewise Linear Characteristics

In piecewise linear model, V_F and r_d are considered

Piecewise Linear Model (Complete Model)

- To construct the piecewise linear characteristic, the forward voltage drop V_F is first marked on the horizontal axis.
- Then, from V_F , a straight line is drawn with a slope equal to the reciprocal of diode dynamic resistance.



Piecewise Linear Characteristic – Numerical Example

Construct the piecewise linear characteristic for a silicon diode which has a 0.25Ω dynamic resistance and a 200 mA maximum forward current.

Solution:

Given $r_d = 0.25 \Omega$ and $I_{F(max)} = 200 \text{ mA}$

Also given that it is a silicon diode. Hence $V_F = 0.7 \text{ V}$

Piecewise Linear Characteristic – Numerical Example

Plot point A on the horizontal axis at $V_F = 0.7\text{ V}$

We know that, $r_d = \frac{\Delta V_F}{\Delta I_F}$

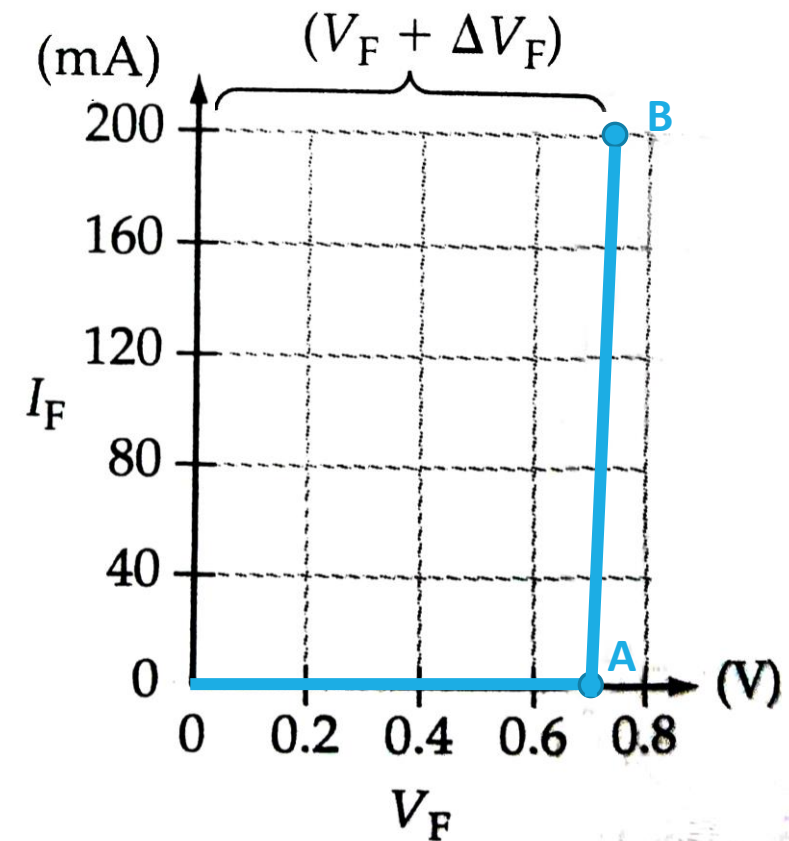
$$\begin{aligned}\text{Hence } \Delta V_F &= \Delta I_F \times r_d \\ &= 200\text{ mA} \times 0.25\ \Omega \\ &= 0.05\text{ V}\end{aligned}$$

Plot point B at

$I_F = 200\text{ mA}$ and

$$V_F = (V_F + \Delta V_F) = (0.7 + 0.05)\text{ V} = 0.75\text{ V}$$

Join points A and B.



Diode Circuits – Numerical Example 1

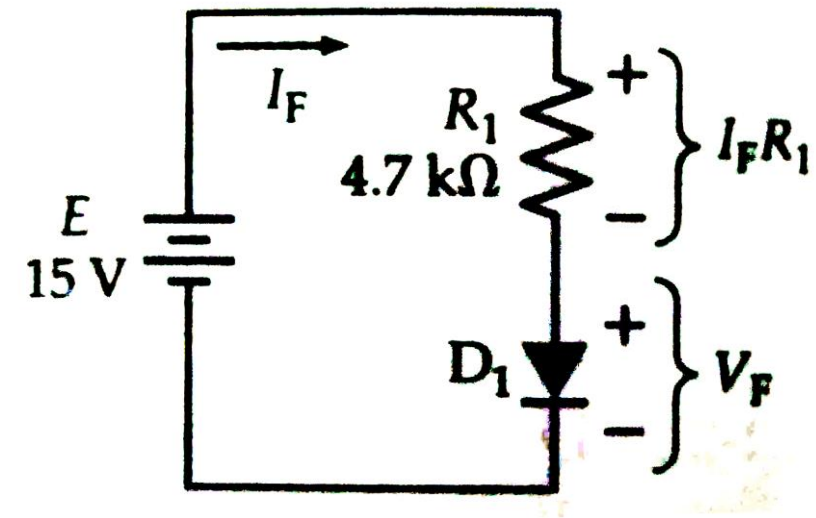
A silicon diode is used in the circuit shown in the figure. Calculate the diode current.

Solution:

Given that it is a silicon diode. Hence $V_F = 0.7 V$

Using KVL, $E = I_F R_1 + V_F$

$$\begin{aligned}\text{Or } I_F &= \frac{E - V_F}{R_1} \\ &= \frac{15 V - 0.7 V}{4.7 k\Omega} \\ &= 3.04 mA\end{aligned}$$



Diode Circuits – Numerical Example 2

Calculate I_F for the diode circuit in the figure assuming that the diode has $V_F = 0.7V$ and $r_d = 0$. Then recalculate the current taking $r_d = 0.25 \Omega$.

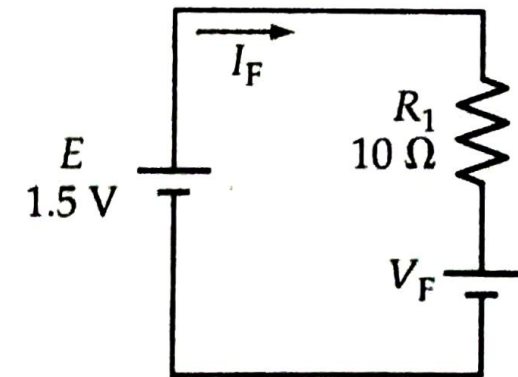
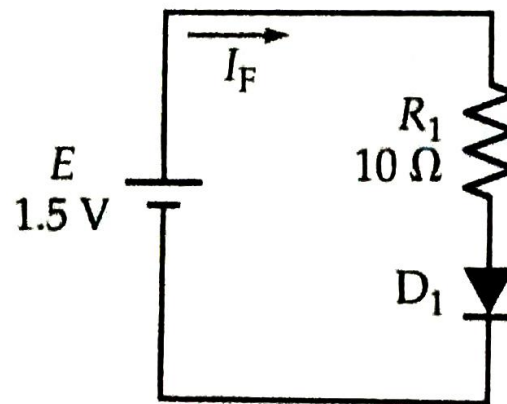
Solution:

Given $V_F = 0.7 V$ and $r_d = 0$

Using diode equivalent circuit,

$$E = I_F R_1 + V_F$$

$$\begin{aligned} \text{Or } I_F &= \frac{E - V_F}{R_1} = \frac{1.5 V - 0.7 V}{10 \Omega} \\ &= 80 \text{ mA} \end{aligned}$$



Diode replaced
with voltage cell

Diode Circuits – Numerical Example 2

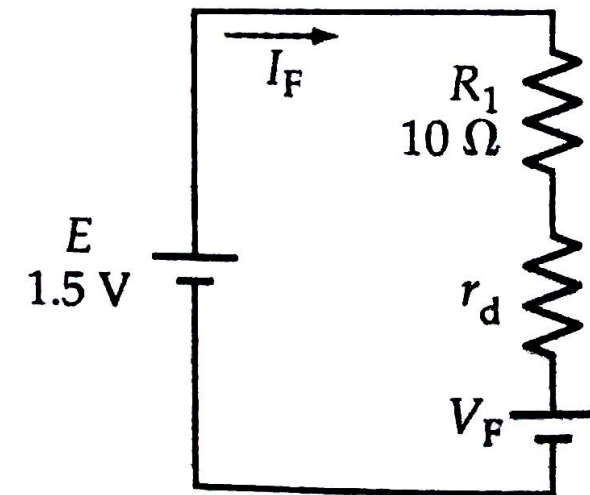
Now consider $r_d = 0.25 \Omega$,

Using diode equivalent circuit,

$$E = I_F R_1 + I_F r_d + V_F$$

$$E = I_F (R_1 + r_d) + V_F$$

$$\begin{aligned} \text{Or } I_F &= \frac{E - V_F}{R_1 + r_d} = \frac{1.5 \text{ V} - 0.7 \text{ V}}{10 \Omega + 0.25 \Omega} \\ &= 78 \text{ mA} \end{aligned}$$



Diode replaced
with r_d and V_F

Diode Circuits – Numerical Example 3

Find the value of the series resistance R required to drive a forward current of 1.25 mA through a Germanium diode from a 4.5 V battery. Write the circuit diagram showing all the values.

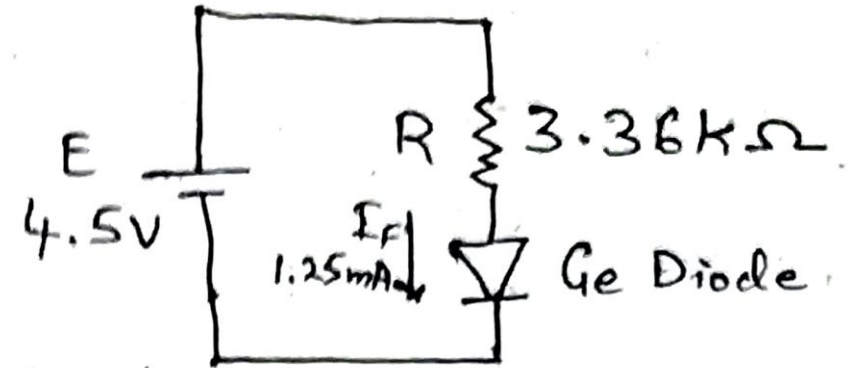
Solution:

Given $I_F = 1.25 \text{ mA}$, $E = 4.5 \text{ V}$

Also that it is a germanium diode. Hence $V_F = 0.3 \text{ V}$

Using KVL, $E = I_F R + V_F$

$$\begin{aligned} \text{Or } R &= \frac{E - V_F}{I_F} = \frac{4.5 \text{ V} - 0.3 \text{ V}}{1.25 \text{ mA}} \\ &= 3.36 \text{ k}\Omega \end{aligned}$$



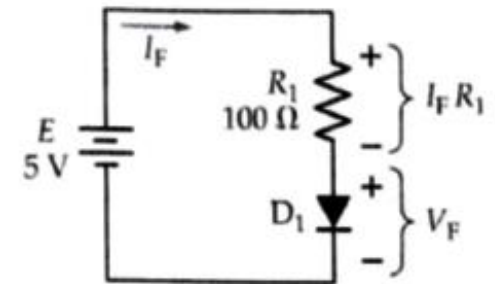
DC Load Line Analysis

DC Load Line

- A *dc load line* is a straight line that illustrates all dc conditions that could exist within the circuit.
- It is drawn on the diode forward characteristics.
- Consider a circuit shown in the figure.
- Using KVL, we can write

$$E = I_F R_1 + V_F \longrightarrow (1)$$

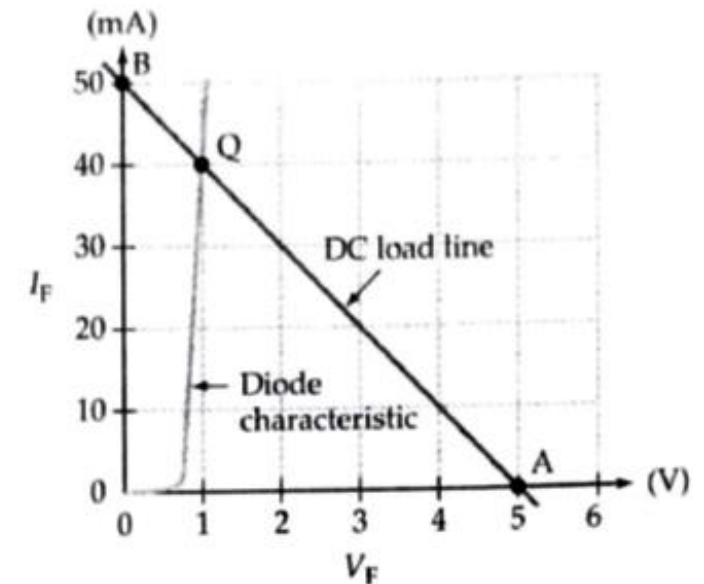
- DC load line can be drawn by obtaining points A and B.



(a) Diode-resistor series circuit

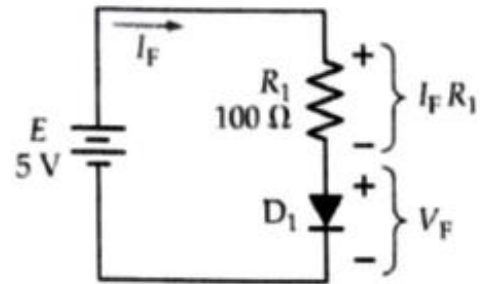
DC Load Line

- At point A, $I_F = 0$. Using this in Eqn. (1), we get $V_F = E$
- At point B, $V_F = 0$. Using this in Eqn. (1), we get $I_F = \frac{E}{R_1}$
- After obtaining points A and B, mark them and join using a straight line.

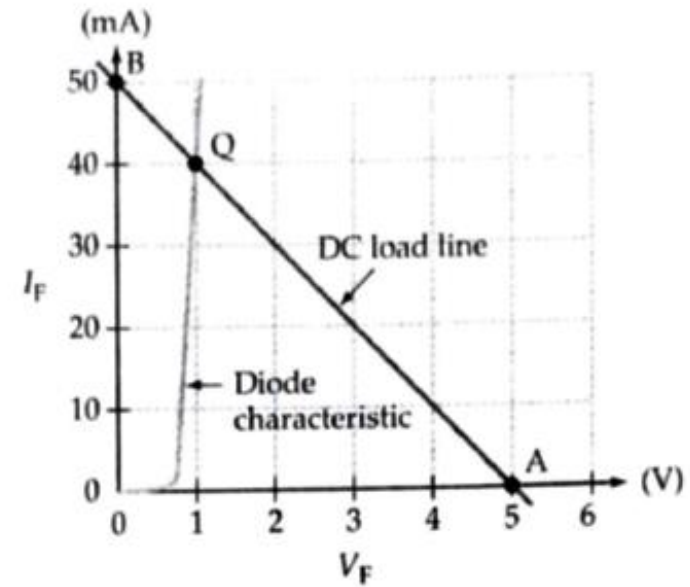


(b) Plotting the dc load line on the diode characteristics

DC Load Line



(a) Diode-resistor series circuit



(b) Plotting the dc load line on the diode characteristics

Q-Point

- The *quiescent point* or *Q-Point* is the only point on the dc load line where the diode voltage and current are compatible with the circuit conditions.
 - It is also called *dc bias point*.
- It is the point where the dc load line intersects the diode forward characteristic.

Rectification

Rectification

- *Rectification* is the process of converting alternating current (ac) to direct current (dc).
- Since semiconductor diodes conduct current in the forward direction and block current in the other direction, they can be used for rectification.
- A *rectifier* is a circuit which converts alternating current (ac) into direct current (dc).
- Rectifiers are found in all dc power supplies that operate from an ac voltage source.
 - A power supply is an essential part of each electronic system from the simplest to the most complex.

Rectification



Why do we need Rectification?

- The source available to us is 230 V, 50 Hz *ac* power supply.
- However, most of the electronic circuits such as amplifiers, oscillators, etc. require a dc voltage in the range of 5 V to 25 V for their proper operation.
- Hence, it is essential to convert *ac* to *dc*.

Types of Rectifiers

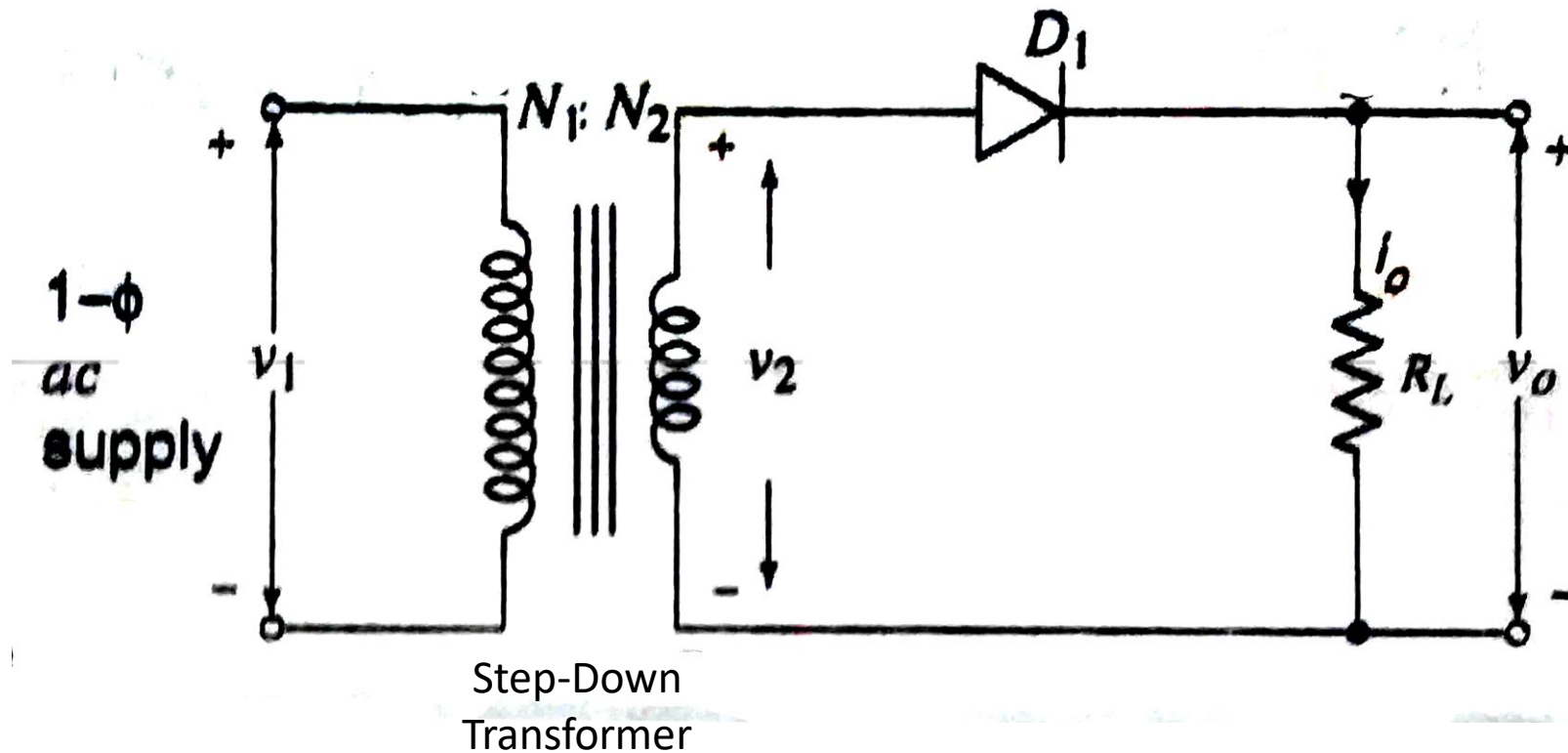
- Half-Wave Rectifier
- Full-Wave Rectifier
 - Full-Wave Rectifier using two diodes and a centre-tapped transformer
 - Full-Wave Bridge Rectifier using four diodes

Half-Wave Rectifier

- A *Half-Wave Rectifier (HWR)* is a circuit which converts only one half-cycle of the input ac to pulsating dc.

Half-Wave Rectifier

Circuit Diagram



Half-Wave Rectifier

- The half-wave rectifier consists of a single diode as shown.
- A step-down transformer is used to reduce the available ac voltage to the required level.
- Resistor R_L is the load resistance which consumes power from the rectifier.

Half-Wave Rectifier

Operation

- Consider the single phase ac input signal given by

$$v_1 = V_m \sin \omega t \quad (1)$$

- We have transformer turns ratio

$$\frac{N_1}{N_2} = \frac{v_1}{v_2}$$

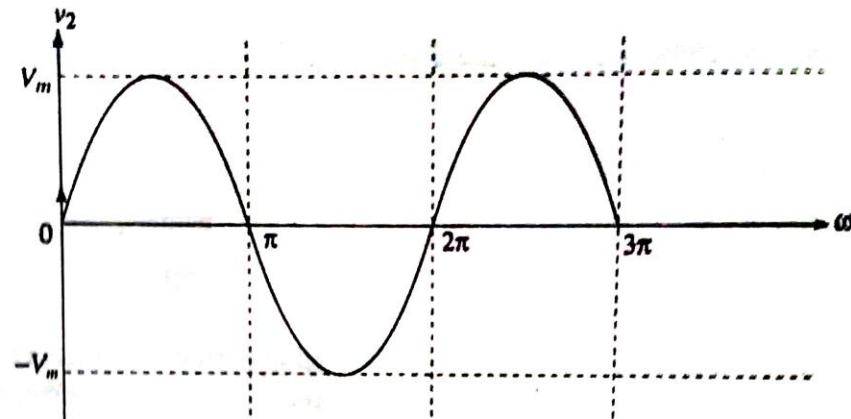
- Rearranging, we can write transformer secondary voltage as

$$v_2 = \frac{N_2}{N_1} V_m \sin \omega t \quad (2)$$

Half-Wave Rectifier

- For simplicity, consider $N_1 = N_2$.
- Then, Eqn. (2) can be written as

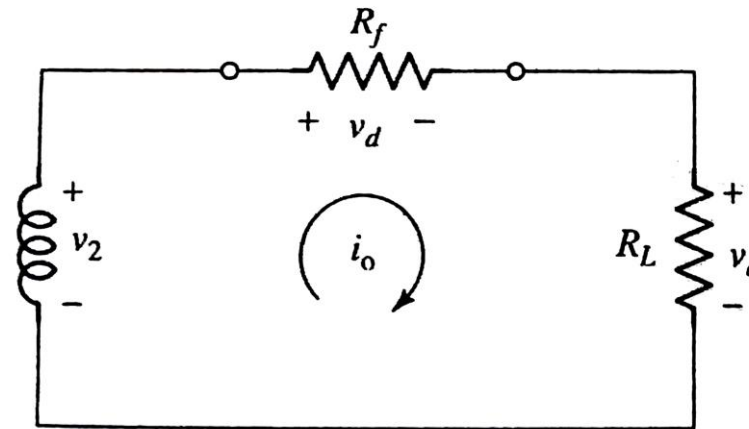
$$v_2 = V_m \sin \omega t \quad (3)$$



Half-Wave Rectifier

(i) During positive half-cycle of ac supply ($0 \leq \omega t \leq \pi$)

- The diode D_1 is forward biased and hence it conducts.
- The conducting diode can be replaced by its forward resistance R_f and the equivalent circuit can be drawn as shown.



Half-Wave Rectifier

- From the circuit,

$$i_o = \frac{v_2}{R_f + R_L}$$

- Using $v_2 = V_m \sin \omega t$, (from Eqn. (3))

$$i_o = \frac{V_m \sin \omega t}{R_f + R_L}$$

- We can write

$$i_o = I_m \sin \omega t \quad (4)$$

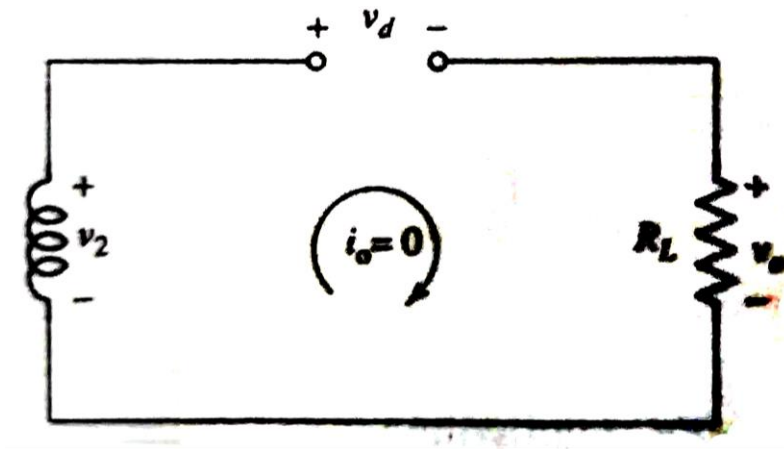
where $I_m = \frac{V_m}{R_f + R_L}$ is the peak value of load current.

- The output voltage is then $v_o = i_o R_L$

Half-Wave Rectifier

(ii) During negative half-cycle of ac supply ($\pi \leq \omega t \leq 2\pi$)

- The diode D_1 is reverse biased and hence it does not conduct.
- The non-conducting diode can be replaced by an open circuit as shown.
- The current i_o is zero and as a result, $v_o = 0$.



From the circuit,

$$i_o = 0 \quad (5)$$

Half-Wave Rectifier

- Using Eqns. (4) and (5), we can write

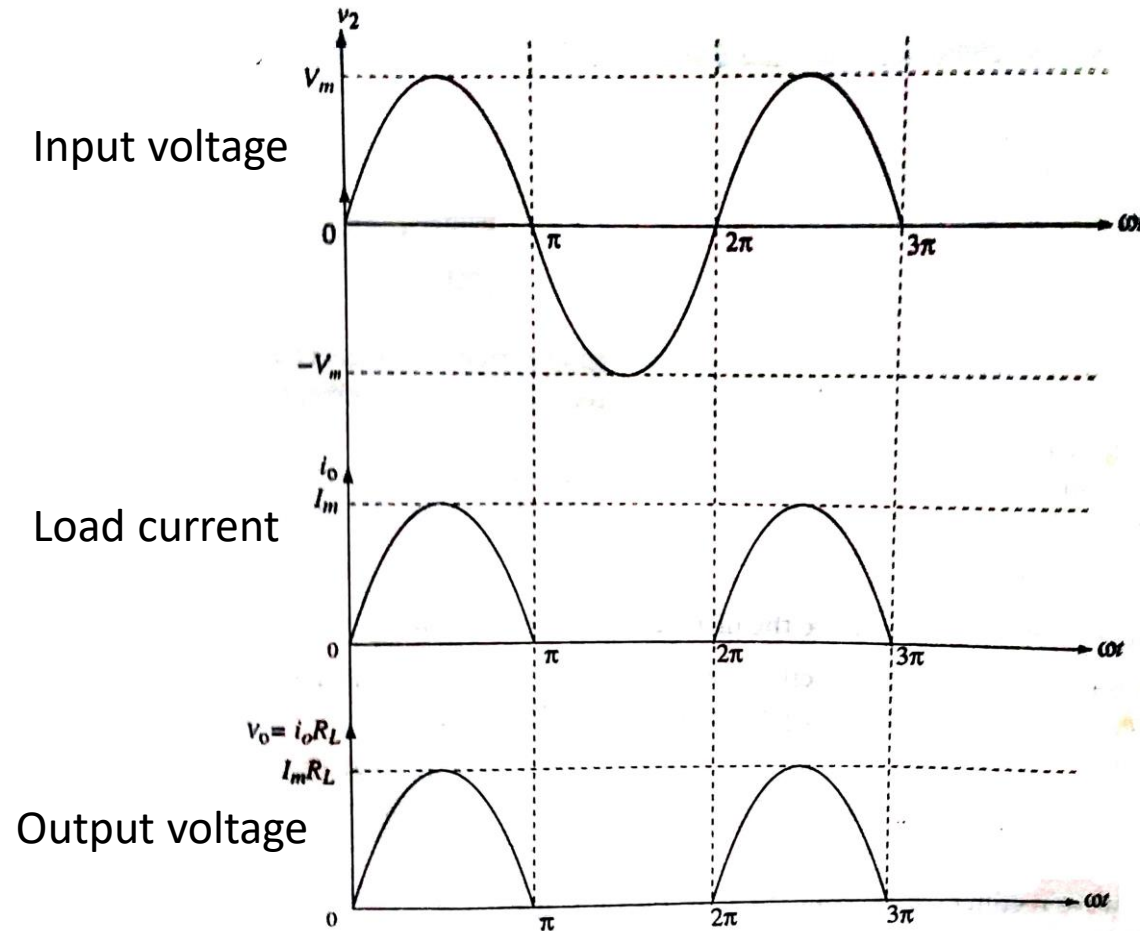
$$i_0 = \begin{cases} I_m \sin \omega t & ; 0 \leq \omega t \leq \pi \\ 0 & ; \pi \leq \omega t \leq 2\pi \end{cases} \quad (6)$$

where $I_m = \frac{V_m}{R_f + R_L}$ is the peak value of load current.

Note: If the diode is ideal, then $R_f = 0$. Then $I_m = \frac{V_m}{R_L}$ (for an ideal diode).

Half-Wave Rectifier

Waveforms



Average or DC Load Current (I_{dc})

$$\begin{aligned} I_{dc} &= \frac{\text{Area under one cycle of } i_o}{\text{Period of } i_o} \\ &= \frac{\int_0^{2\pi} i_o d\omega t}{2\pi} \\ &= \frac{1}{2\pi} \left[\int_0^{\pi} I_m \sin \omega t d\omega t + \int_{\pi}^{2\pi} 0 d\omega t \right] \\ &= \frac{I_m}{2\pi} [-\cos \omega t]_0^{\pi} \end{aligned}$$

Average or DC Load Current (I_{dc})

$$\begin{aligned} I_{dc} &= \frac{I_m}{2\pi} [-\cos \pi - (-\cos 0)] \\ &= \frac{I_m}{2\pi} [-(-1) - (-1)] \\ &= \frac{I_m}{2\pi} [2] \end{aligned}$$

$$I_{dc} = \frac{I_m}{\pi}$$

Average or DC Output Voltage (V_{dc})

$$V_{dc} = I_{dc}R_L$$

$$= \frac{I_m}{\pi} R_L$$

$$= \frac{1}{\pi} \left[\frac{V_m}{R_f + R_L} \right] R_L$$

$$\therefore I_m = \frac{V_m}{R_f + R_L}$$

$$V_{dc} = \frac{V_m}{\pi} \frac{R_L}{R_f + R_L}$$

Average or DC Output Voltage (V_{dc})

Dividing numerator and denominator by R_L ,

$$V_{dc} = \frac{(V_m/\pi)}{1 + (R_f/R_L)}$$

Note: If the diode is ideal, then $R_f = 0$. Then $V_{dc} = \frac{V_m}{\pi}$ (for an ideal diode).

RMS or AC Load Current (I_{rms})

RMS = Root Mean Square

$$\begin{aligned} I_{rms} &= \sqrt{\frac{\text{Area under one cycle of } i_o^2}{\text{Period of } i_o^2}} \\ &= \sqrt{\frac{\int_0^{2\pi} i_o^2 d\omega t}{2\pi}} \\ &= \sqrt{\frac{1}{2\pi} \left\{ \int_0^{\pi} I_m^2 \sin^2 \omega t d\omega t + \int_{\pi}^{2\pi} 0 d\omega t \right\}} \end{aligned}$$

RMS or AC Load Current (I_{rms})

$$= I_m \sqrt{\frac{1}{2\pi} \int_0^\pi \sin^2 \omega t \, d\omega t}$$

$$= I_m \sqrt{\frac{1}{2\pi} \int_0^\pi \left[\frac{1 - \cos 2\omega t}{2} \right] d\omega t}$$

$$\because \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$$

$$= I_m \sqrt{\frac{1}{4\pi} \left[\int_0^\pi 1 \, d\omega t - \int_0^\pi \cos 2\omega t \, d\omega t \right]}$$

RMS or AC Load Current (I_{rms})

$$\begin{aligned} &= \frac{I_m}{2} \sqrt{\frac{1}{\pi} \left\{ [\omega t]_0^\pi - \left[\frac{\sin 2\omega t}{2} \right]_0^\pi \right\}} \\ &= \frac{I_m}{2} \sqrt{\frac{1}{\pi} \left\{ (\pi - 0) - \frac{1}{2} (\sin 2\pi - \sin 0) \right\}} \\ &= \frac{I_m}{2} \sqrt{\frac{1}{\pi} (\pi)} \end{aligned}$$

RMS or AC Load Current (I_{rms})

$$I_{rms} = \frac{I_m}{2}$$

RMS or AC Output Voltage (V_{rms})

$$V_{rms} = I_{rms} R_L$$

$$= \frac{I_m}{2} R_L$$

$$= \frac{1}{2} \left[\frac{V_m}{R_f + R_L} \right] R_L$$

$$\therefore I_m = \frac{V_m}{R_f + R_L}$$

$$V_{rms} = \frac{V_m}{2} \frac{R_L}{R_f + R_L}$$

RMS or AC Output Voltage (V_{rms})

Dividing numerator and denominator by R_L ,

$$V_{rms} = \frac{(V_m/2)}{1 + (R_f/R_L)}$$

Note: If the diode is ideal, then $R_f = 0$. Then $V_{rms} = \frac{V_m}{2}$ (for an ideal diode).

Rectification Efficiency (η_r)

- Rectification efficiency is defined as the ratio of the dc output power to the ac input power supplied to the rectifier.
- It is given by,

$$\eta_r = \frac{P_{dc}}{P_i} \quad (1)$$

where P_{dc} is the dc output power of the rectifier and P_i is the ac input power to the rectifier.

Rectification Efficiency (η_r)

- The dc output power is given by

$$\begin{aligned} P_{dc} &= I_{dc}^2 R_L \\ &= \left[\frac{I_m}{\pi} \right]^2 R_L \end{aligned} \quad \because I_{dc} = \frac{I_m}{\pi} \text{ for a HWR}$$
$$P_{dc} = \frac{I_m^2 R_L}{\pi^2} \quad (2)$$

Rectification Efficiency (η_r)

- The ac input power is given by

$$\begin{aligned} P_i &= I_{rms}^2 [R_f + R_L] \\ &= \left[\frac{I_m}{2} \right]^2 [R_f + R_L] \\ P_i &= \frac{I_m^2}{4} [R_f + R_L] \end{aligned}$$

$$\because I_{rms} = \frac{I_m}{2} \text{ for a HWR}$$

(3)

Rectification Efficiency (η_r)

- Using Eqns. (2) and (3) in (1),

$$\eta_r = \frac{\frac{I_m^2 R_L}{\pi^2}}{\frac{I_m^2}{4} [R_f + R_L]}$$
$$= \frac{4 R_L}{\pi^2 R_f + R_L}$$

$$\eta_r = \frac{0.405 R_L}{R_f + R_L}$$

Rectification Efficiency (η_r)

$$\eta_r = \frac{0.405}{1 + (R_f/R_L)}$$

$$\% \eta_r = \frac{0.405}{1 + (R_f/R_L)} \times 100\%$$

$$\% \eta_r = \frac{40.5}{1 + (R_f/R_L)} \%$$

Note: Maximum efficiency can be achieved for an ideal diode ($R_f = 0$)

$$\% \eta_{r(max)} = 40.5\%$$

Rectification Efficiency (η_r)

Note: We have $\% \eta_{r(max)} = 40.5\%$.

That means,

$$\eta_{r(max)} = \frac{P_{dc}}{P_i} = 0.405$$

$$P_{dc} = 0.405P_i$$

- The dc output power is only 40.5% of the ac input power.
 - The remaining 59.5% of the ac input power goes unused.
- Hence, half-wave rectifier has a very poor rectification efficiency.

Ripple Factor (γ)

- Ripple factor is the ratio of rms value of ac component present in the rectified output to the dc component of the rectified output.
- It is given by,

$$\gamma = \frac{V_{ac}}{V_{dc}} \quad (1)$$

where V_{ac} is the rms value of ac component present in the rectified output

and V_{dc} is the dc component of the rectified output.

Ripple Factor (γ)

- The total power output is the sum of powers of dc and ac components.

$$P_{total} = P_{dc} + P_{ac}$$

$$\frac{V_{rms}^2}{R_L} = \frac{V_{dc}^2}{R_L} + \frac{V_{ac}^2}{R_L}$$

$$V_{rms}^2 = V_{dc}^2 + V_{ac}^2$$

Ripple Factor (γ)

Dividing throughout by V_{dc}^2 , we get

$$\frac{V_{rms}^2}{V_{dc}^2} = \frac{V_{dc}^2}{V_{dc}^2} + \frac{V_{ac}^2}{V_{dc}^2}$$

$$\left[\frac{V_{rms}}{V_{dc}} \right]^2 = 1 + \left[\frac{V_{ac}}{V_{dc}} \right]^2$$

$$\left[\frac{V_{ac}}{V_{dc}} \right]^2 = \left[\frac{V_{rms}}{V_{dc}} \right]^2 - 1$$

Ripple Factor (γ)

$$\frac{V_{ac}}{V_{dc}} = \sqrt{\left[\frac{V_{rms}}{V_{dc}}\right]^2 - 1}$$

Using Eqn. (1),

$$\gamma = \sqrt{\left[\frac{V_{rms}}{V_{dc}}\right]^2 - 1} \quad (2)$$

Ripple Factor (γ)

In a half-wave rectifier, $V_{rms} = \frac{(V_m/2)}{1+(R_f/R_L)}$ and $V_{dc} = \frac{(V_m/\pi)}{1+(R_f/R_L)}$

$$\gamma = \sqrt{\left[\frac{\frac{(V_m/2)}{1+(R_f/R_L)}}{\frac{(V_m/\pi)}{1+(R_f/R_L)}} \right]^2 - 1}$$

$$\gamma = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1}$$

Ripple Factor (γ)

$$\gamma = \sqrt{\frac{\pi^2}{4} - 1}$$

$$\gamma = 1.21$$

Ripple Factor (γ)

Note: We have $\gamma = 1.21$.

That means,

$$\gamma = \frac{V_{ac}}{V_{dc}} = 1.21$$
$$V_{ac} = 1.21V_{dc}$$

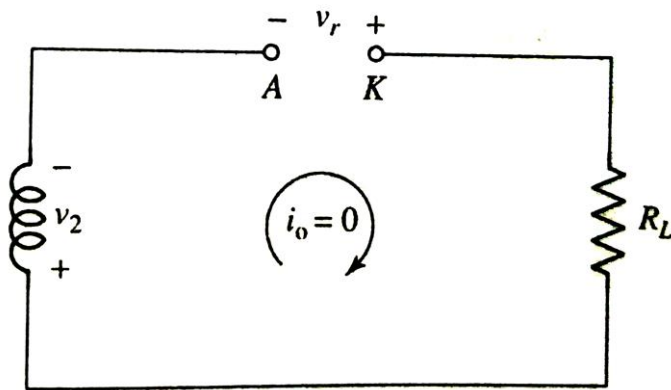
- The ac or ripple component is 121% of the dc component, i.e., ac component is greater than dc component.
- Hence, half-wave rectifier is not recommended for practical applications.

Peak Inverse Voltage (PIV)

- Peak Inverse Voltage (PIV) is the maximum reverse voltage to which the diode can be subjected.
- If the applied reverse voltage across the diode is greater than its PIV rating, the reverse breakdown of the diode which causes a permanent damage to the diode.

Peak Inverse Voltage (PIV)

- Consider the equivalent circuit of a half-wave rectifier when the diode is not conducting.



v_r is the instantaneous reverse voltage across the diode.

- From the circuit, since $i_o = 0$,

$$v_r = v_2$$

$$v_r = V_m \sin \omega t$$

- Now,

$$PIV = v_{r(max)}$$

$$PIV = V_m$$

Half-Wave Rectifier – Numerical Example 1

In a half wave rectifier, the input is from 30 V transformer. The load and diode forward resistances are 100 Ω and 10 Ω respectively. Calculate the I_{dc} , I_{rms} , P_{dc} , P_i , η , PIV and ripple factor.

Solution:

Given $V_2 = 30\text{ V}$, $R_L = 100\ \Omega$, $R_f = 10\ \Omega$

The given V_2 is the rms value of the input and we know that $V_{rms} = \frac{V_m}{\sqrt{2}}$

$$\begin{aligned}\text{Hence, } V_m &= \sqrt{2}V_2 \\ &= \sqrt{2} \times 30\text{ V}\end{aligned}$$

$$V_m = 42.426\text{ V}$$

Half-Wave Rectifier – Numerical Example 1

$$\begin{aligned}\text{Then, } I_m &= \frac{V_m}{R_f + R_L} \\ &= \frac{42.426 \text{ V}}{10 \Omega + 100 \Omega} \\ I_m &= 385.69 \text{ mA}\end{aligned}$$

DC Load Current

$$\begin{aligned}I_{dc} &= \frac{I_m}{\pi} \\ &= \frac{385.69 \text{ mA}}{\pi} \\ I_{dc} &= 122.77 \text{ mA}\end{aligned}$$

RMS Load Current

$$\begin{aligned}I_{rms} &= \frac{I_m}{2} \\ &= \frac{385.69 \text{ mA}}{2} \\ I_{rms} &= 192.845 \text{ mA}\end{aligned}$$

DC Output Voltage

$$\begin{aligned}V_{dc} &= I_{dc} R_L \\ &= 122.77 \text{ mA} \times 100 \Omega \\ V_{dc} &= 12.277 \text{ V}\end{aligned}$$

RMS Output Voltage

$$\begin{aligned}V_{rms} &= I_{rms} R_L \\ &= 192.845 \text{ mA} \times 100 \Omega \\ V_{rms} &= 19.2845 \text{ V}\end{aligned}$$

Half-Wave Rectifier – Numerical Example 1

AC Input Power

$$\begin{aligned}P_i &= I_{rms}^2 [R_f + R_L] \\ &= (192.845m)^2 \times (10 + 100) \\ P_i &= 4.091 \text{ W}\end{aligned}$$

DC Output Power

$$\begin{aligned}P_{dc} &= I_{dc}^2 R_L \\ &= (122.77m)^2 \times 100 \\ P_{dc} &= 1.507 \text{ W}\end{aligned}$$

Rectification Efficiency

$$\begin{aligned}\% \eta_r &= \frac{P_{dc}}{P_i} \times 100 \% \\ &= \frac{1.507}{4.091} \times 100 \% \\ \% \eta_r &= 36.84 \%\end{aligned}$$

Half-Wave Rectifier – Numerical Example 1

Ripple Factor

$$\begin{aligned}\gamma &= \sqrt{\left[\frac{V_{rms}}{V_{dc}}\right]^2 - 1} \\ &= \sqrt{\left(\frac{19.2845}{12.277}\right)^2 - 1} \\ \gamma &= 1.21\end{aligned}$$

Peak Inverse Voltage

$$\begin{aligned}PIV &= V_m \\ PIV &= 42.426\text{ V}\end{aligned}$$

Half-Wave Rectifier – Numerical Example 2

The input to a half wave rectifier is given through a 10:1 transformer from a supply given by $230 \sin 314t \text{ V}$. If $R_f = 50 \Omega$ and $R_L = 500 \Omega$, determine DC load voltage, RMS load voltage, rectification efficiency, DC power delivered to the load.

Solution:

$$\text{Given } v_1 = 230 \sin 314t \text{ V}$$

$$N_1 : N_2 = 10 : 1$$

$$R_f = 50 \Omega$$

$$R_L = 500 \Omega$$

Half-Wave Rectifier – Numerical Example 2

We know that, $\frac{v_1}{v_2} = \frac{N_1}{N_2}$

Therefore, $v_2 = \frac{N_2}{N_1} v_1$
 $= \frac{1}{10} \times 230 \sin 314t \text{ V}$

$$v_2 = 23 \sin 314t \text{ V}$$

Comparing this with $V_m \sin \omega t$,

We have $V_m = 23 \text{ V}$ and $\omega = 314 \text{ rad/s}$

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{314}{2\pi}$$

$$f = 49.97 \text{ Hz} \cong 50 \text{ Hz}$$

Half-Wave Rectifier – Numerical Example 2

$$\begin{aligned}\text{Then, } I_m &= \frac{V_m}{R_f + R_L} \\ &= \frac{23 \text{ V}}{50 \Omega + 500 \Omega} \\ I_m &= 41.818 \text{ mA}\end{aligned}$$

DC Load Current

$$\begin{aligned}I_{dc} &= \frac{I_m}{\pi} \\ &= \frac{41.818 \text{ mA}}{\pi} \\ I_{dc} &= 13.31 \text{ mA}\end{aligned}$$

RMS Load Current

$$\begin{aligned}I_{rms} &= \frac{I_m}{2} \\ &= \frac{41.818 \text{ mA}}{2} \\ I_{rms} &= 20.909 \text{ mA}\end{aligned}$$

DC Load Voltage

$$\begin{aligned}V_{dc} &= I_{dc} R_L \\ &= 13.31 \text{ mA} \times 500 \Omega \\ V_{dc} &= 6.655 \text{ V}\end{aligned}$$

RMS Load Voltage

$$\begin{aligned}V_{rms} &= I_{rms} R_L \\ &= 20.909 \text{ mA} \times 500 \Omega \\ V_{rms} &= 10.45 \text{ V}\end{aligned}$$

Half-Wave Rectifier – Numerical Example 2

AC Input Power

$$\begin{aligned}P_i &= I_{rms}^2 [R_f + R_L] \\ &= (20.909m)^2 \times (50 + 500) \\ P_i &= 240.452 \text{ mW}\end{aligned}$$

DC Power delivered to the load

$$\begin{aligned}P_{dc} &= I_{dc}^2 R_L \\ &= (13.31m)^2 \times 500 \\ P_{dc} &= 88.578 \text{ mW}\end{aligned}$$

Rectification Efficiency

$$\begin{aligned}\% \eta_r &= \frac{P_{dc}}{P_i} \times 100 \% \\ &= \frac{88.578 \text{ mW}}{240.452 \text{ mW}} \times 100 \% \\ \% \eta_r &= 36.84 \%\end{aligned}$$

Half-Wave Rectifier – Numerical Example 3

A half wave rectifier is fed from a supply 230 V, 50 Hz with a step-down transformer of ratio 3:1. Resistive load connected is 10 k Ω . The diode forward resistance is 75 Ω and transformer secondary is 10 Ω . Calculate the DC load current, DC load voltage, efficiency and ripple factor.

Solution:

Given $V_1 = 230\text{ V}$, $f = 50\text{ Hz}$, $N_1:N_2 = 3:1$

$$R_L = 10\text{ k}\Omega = 10000\ \Omega$$

$$R_f = 75\ \Omega$$

$$R_s = 10\ \Omega$$

Half-Wave Rectifier – Numerical Example 3

We know that, $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$$\begin{aligned}\text{Therefore, } V_2 &= \frac{N_2}{N_1} V_1 \\ &= \frac{1}{3} \times 230 \text{ V}\end{aligned}$$

$$V_2 = 76.67 \text{ V}$$

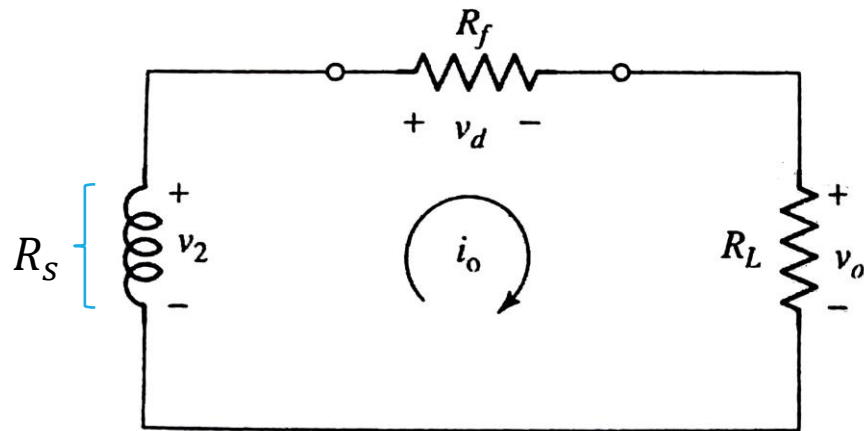
This value of V_2 is the rms value and the peak value can be found using

$$\begin{aligned}V_m &= \sqrt{2} V_2 \\ &= \sqrt{2} \times 76.67 \text{ V}\end{aligned}$$

$$V_m = 108.42 \text{ V}$$

Half-Wave Rectifier – Numerical Example 3

Here, the resistance of the transformer secondary R_S is given. So it should be considered in series with R_f and R_L .



From the circuit,

$$i_o = \frac{v_2}{R_S + R_f + R_L}$$

$$i_o = \frac{V_m \sin \omega t}{R_S + R_f + R_L}$$

We can write

$$i_o = I_m \sin \omega t$$

where $I_m = \frac{V_m}{R_S + R_f + R_L}$ is the peak value of load current.

Half-Wave Rectifier – Numerical Example 3

$$\begin{aligned}\text{So, } I_m &= \frac{V_m}{R_S + R_f + R_L} \\ &= \frac{108.42 \text{ V}}{10 \Omega + 75 \Omega + 10 \text{ k}\Omega} \\ I_m &= 10.75 \text{ mA}\end{aligned}$$

DC Load Current

$$\begin{aligned}I_{dc} &= \frac{I_m}{\pi} \\ &= \frac{10.75 \text{ mA}}{\pi} \\ I_{dc} &= 3.422 \text{ mA}\end{aligned}$$

RMS Load Current

$$\begin{aligned}I_{rms} &= \frac{I_m}{2} \\ &= \frac{10.75 \text{ mA}}{2}\end{aligned}$$

$$I_{rms} = 5.375 \text{ mA}$$

DC Load Voltage

$$\begin{aligned}V_{dc} &= I_{dc} R_L \\ &= 3.422 \text{ mA} \times 10 \text{ k}\Omega\end{aligned}$$

$$V_{dc} = 34.22 \text{ V}$$

RMS Load Voltage

$$\begin{aligned}V_{rms} &= I_{rms} R_L \\ &= 5.375 \text{ mA} \times 10 \text{ k}\Omega\end{aligned}$$

$$V_{rms} = 53.75 \text{ V}$$

Half-Wave Rectifier – Numerical Example 3

AC Input Power

$$\begin{aligned}P_i &= I_{rms}^2 [R_s + R_f + R_L] \\ &= (5.375m)^2 \times (10 + 75 + 10k) \\ P_i &= 291.36 \text{ mW}\end{aligned}$$

DC Output Power

$$\begin{aligned}P_{dc} &= I_{dc}^2 R_L \\ &= (3.422m)^2 \times 10k \\ P_{dc} &= 117.1 \text{ mW}\end{aligned}$$

Rectification Efficiency

$$\begin{aligned}\% \eta_r &= \frac{P_{dc}}{P_i} \times 100 \% \\ &= \frac{117.1 \text{ mW}}{291.36 \text{ mW}} \times 100 \% \\ \% \eta_r &= 40.19 \%\end{aligned}$$

Half-Wave Rectifier – Numerical Example 3

Ripple Factor

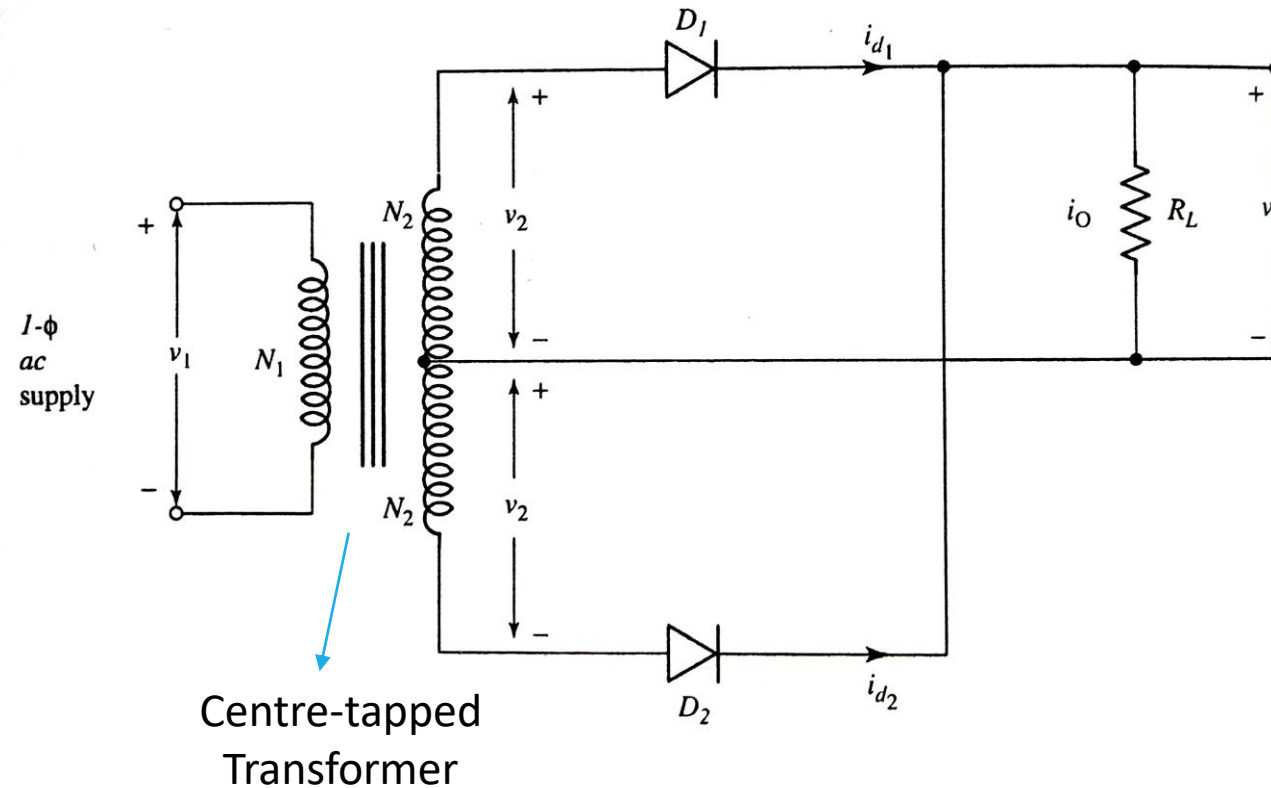
$$\begin{aligned}\gamma &= \sqrt{\left[\frac{V_{rms}}{V_{dc}}\right]^2 - 1} \\ &= \sqrt{\left(\frac{53.75}{34.22}\right)^2 - 1} \\ \gamma &= 1.21\end{aligned}$$

Full-Wave Rectifier

- A *Full-Wave Rectifier (FWR)* is a circuit which converts both the half-cycles of the input ac to pulsating dc.
- A full-wave rectifier can be constructed using two ways:
 - Using two diodes and a centre-tapped transformer
 - Using four diodes (Bridge Rectifier)

Full-Wave Rectifier

Circuit Diagram



Full-Wave Rectifier

- The full-wave rectifier consists of two diodes D_1 and D_2 as shown.
- A step-down transformer is used to reduce the available ac voltage to the required level.
- The transformer is centre-tapped so that two equal voltages are induced at both halves of the transformer secondary.
- Resistor R_L is the load resistance which consumes power from the rectifier.

Full-Wave Rectifier

Operation

- Consider the single phase ac input signal given by

$$v_1 = V_m \sin \omega t \quad (1)$$

- We have transformer turns ratio

$$\frac{N_1}{N_2} = \frac{v_1}{v_2}$$

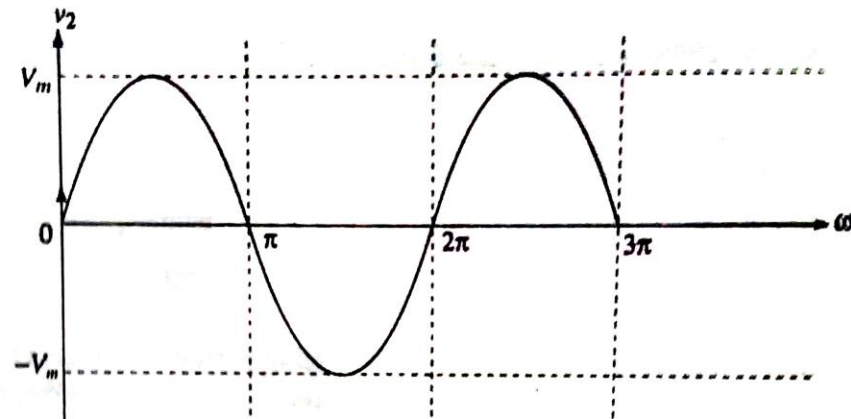
- Rearranging, we can write transformer secondary voltage as

$$v_2 = \frac{N_2}{N_1} V_m \sin \omega t \quad (2)$$

Full-Wave Rectifier

- For simplicity, consider $N_1 = N_2$.
- Then, Eqn. (2) can be written as

$$v_2 = V_m \sin \omega t \quad (3)$$



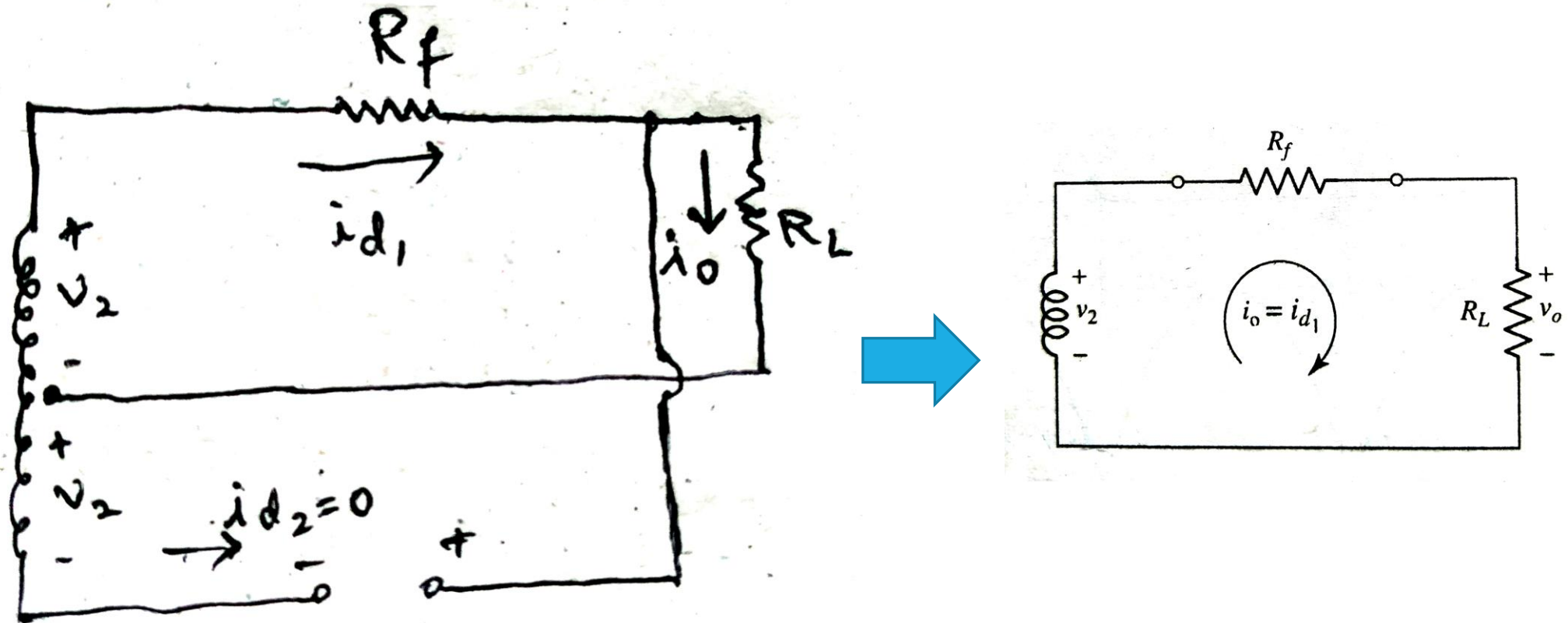
Full-Wave Rectifier

(i) During positive half-cycle of ac supply ($0 \leq \omega t \leq \pi$)

- The diode D_1 is forward biased and hence it conducts, whereas the diode D_2 is reverse biased and hence it remains *off*.
- The conducting diode can be replaced by its forward resistance R_f and the non-conducting diode can be replaced by an open circuit and the equivalent circuit can be drawn as shown.

Full-Wave Rectifier

Equivalent circuit during positive half-cycle



Full-Wave Rectifier

- From the circuit,

$$i_o = \frac{v_2}{R_f + R_L}$$

- Using $v_2 = V_m \sin \omega t$, (from Eqn. (3))

$$i_o = \frac{V_m \sin \omega t}{R_f + R_L}$$

- We can write

$$i_o = I_m \sin \omega t \quad (4)$$

where $I_m = \frac{V_m}{R_f + R_L}$ is the peak value of load current.

- The output voltage is then $v_o = i_o R_L$

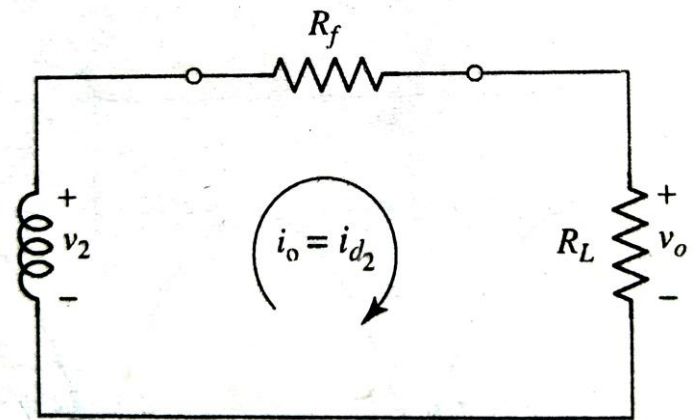
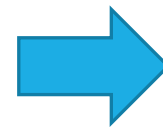
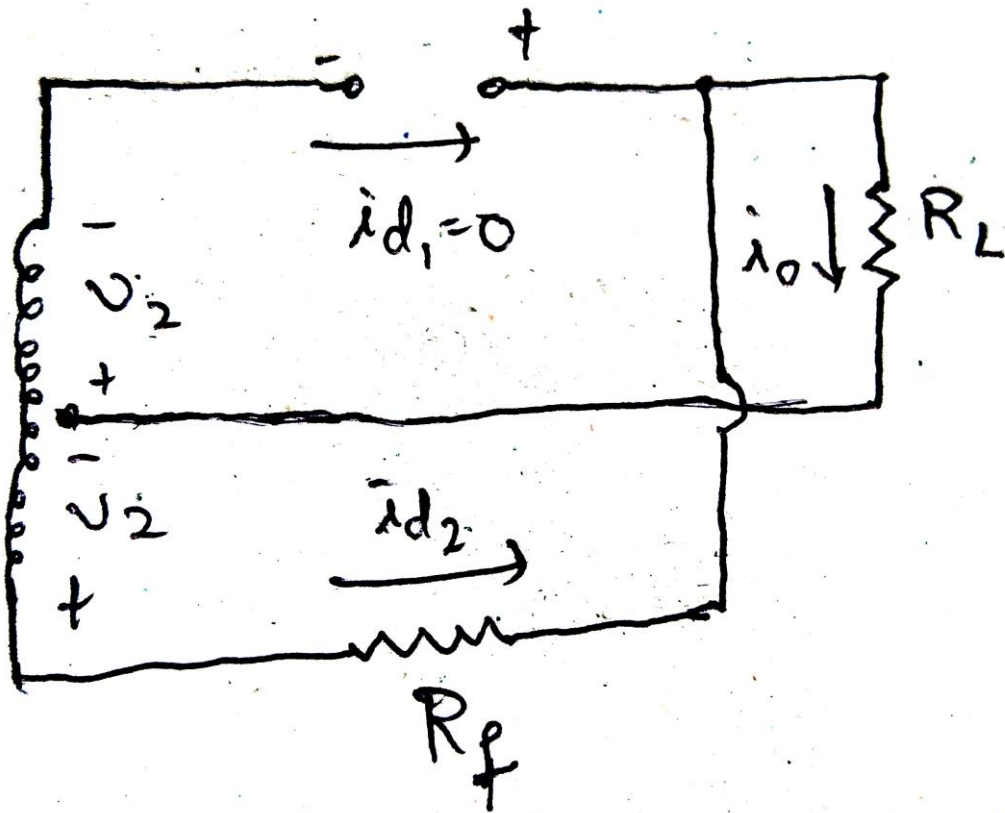
Full-Wave Rectifier

(ii) During negative half-cycle of ac supply ($\pi \leq \omega t \leq 2\pi$)

- The diode D_2 is forward biased and hence it conducts, whereas the diode D_1 is reverse biased and hence it remains *off*.
- The conducting diode can be replaced by its forward resistance R_f and the non-conducting diode can be replaced by an open circuit and the equivalent circuit can be drawn as shown.

Full-Wave Rectifier

Equivalent circuit during negative half-cycle



Full-Wave Rectifier

- As the equivalent circuit is similar, we can write

$$i_o = I_m \sin \omega t \quad (5)$$

where $I_m = \frac{V_m}{R_f + R_L}$ is the peak value of load current.

- The output voltage is then $v_o = i_o R_L$

Full-Wave Rectifier

- Using Eqns. (4) and (5), we can write

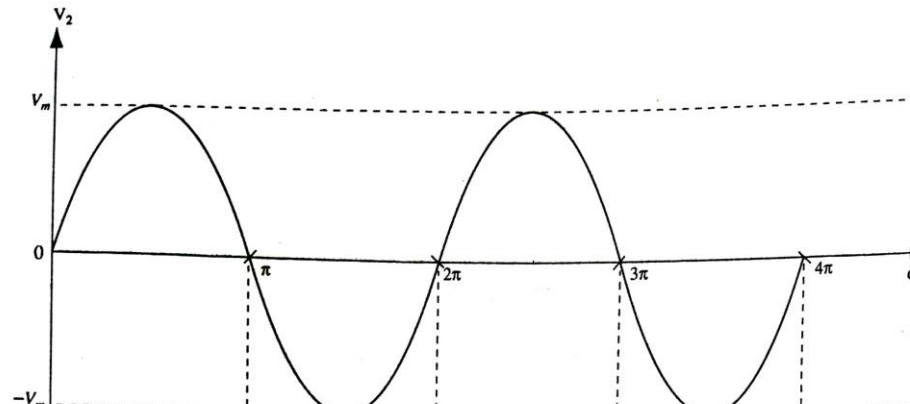
$$\boxed{i_0 = I_m \sin \omega t} \quad (6)$$

where $I_m = \frac{V_m}{R_f + R_L}$ is the peak value of load current.

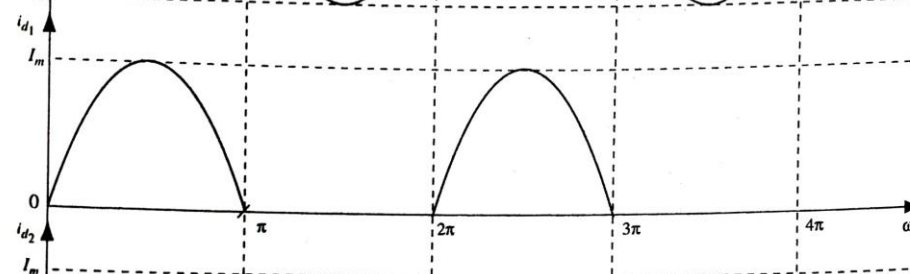
Note: If the diode is ideal, then $R_f = 0$. Then $I_m = \frac{V_m}{R_L}$ (for an ideal diode).

Waveforms

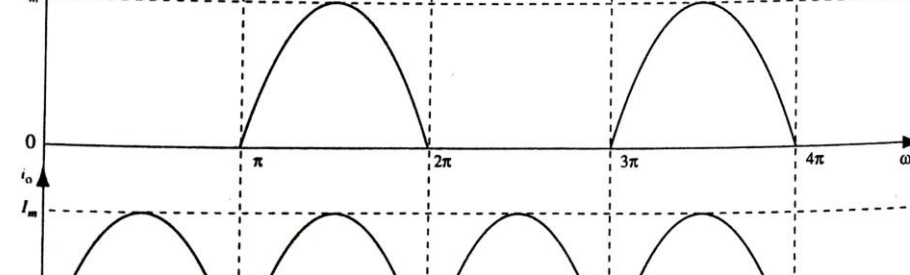
Input voltage



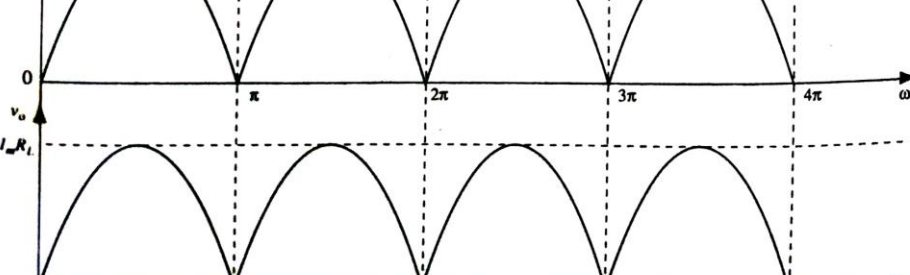
Current through diode D_1



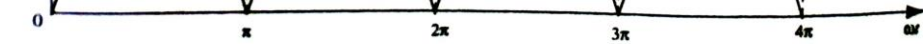
Current through diode D_2



Load current



Output voltage



Average or DC Load Current (I_{dc})

$$\begin{aligned} I_{dc} &= \frac{\text{Area under one cycle of } i_o}{\text{Period of } i_o} \\ &= \frac{\int_0^{\pi} i_o d\omega t}{\pi} \\ &= \frac{1}{\pi} \left[\int_0^{\pi} I_m \sin \omega t d\omega t \right] \\ &= \frac{I_m}{\pi} [-\cos \omega t]_0^{\pi} \end{aligned}$$

Average or DC Load Current (I_{dc})

$$\begin{aligned} I_{dc} &= \frac{I_m}{\pi} [-\cos \pi - (-\cos 0)] \\ &= \frac{I_m}{\pi} [-(-1) - (-1)] \\ &= \frac{I_m}{\pi} [2] \end{aligned}$$

$$I_{dc} = \frac{2I_m}{\pi}$$

Average or DC Output Voltage (V_{dc})

$$V_{dc} = I_{dc}R_L$$

$$= \frac{2I_m}{\pi} R_L$$

$$= \frac{2}{\pi} \left[\frac{V_m}{R_f + R_L} \right] R_L$$

$$\therefore I_m = \frac{V_m}{R_f + R_L}$$

$$V_{dc} = \frac{2V_m}{\pi} \frac{R_L}{R_f + R_L}$$

Average or DC Output Voltage (V_{dc})

Dividing numerator and denominator by R_L ,

$$V_{dc} = \frac{(2V_m/\pi)}{1 + (R_f/R_L)}$$

Note: If the diode is ideal, then $R_f = 0$. Then $V_{dc} = \frac{2V_m}{\pi}$ (for an ideal diode).

RMS or AC Load Current (I_{rms})

RMS = Root Mean Square

$$\begin{aligned} I_{rms} &= \sqrt{\frac{\text{Area under one cycle of } i_o^2}{\text{Period of } i_o^2}} \\ &= \sqrt{\frac{\int_0^\pi i_o^2 d\omega t}{\pi}} \\ &= \sqrt{\frac{1}{\pi} \int_0^\pi I_m^2 \sin^2 \omega t d\omega t} \end{aligned}$$

RMS or AC Load Current (I_{rms})

$$= I_m \sqrt{\frac{1}{\pi} \int_0^{\pi} \sin^2 \omega t \, d\omega t}$$

$$= I_m \sqrt{\frac{1}{\pi} \int_0^{\pi} \left[\frac{1 - \cos 2\omega t}{2} \right] d\omega t}$$

$$\because \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$$

$$= I_m \sqrt{\frac{1}{2\pi} \left[\int_0^{\pi} 1 \, d\omega t - \int_0^{\pi} \cos 2\omega t \, d\omega t \right]}$$

RMS or AC Load Current (I_{rms})

$$= I_m \sqrt{\frac{1}{2\pi} \left\{ [\omega t]_0^\pi - \left[\frac{\sin 2\omega t}{2} \right]_0^\pi \right\}}$$

$$= I_m \sqrt{\frac{1}{2\pi} \left\{ (\pi - 0) - \frac{1}{2} (\sin 2\pi - \sin 0) \right\}}$$

$$= I_m \sqrt{\frac{1}{2\pi} (\pi)}$$

RMS or AC Load Current (I_{rms})

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

RMS or AC Output Voltage (V_{rms})

$$V_{rms} = I_{rms} R_L$$

$$= \frac{I_m}{\sqrt{2}} R_L$$

$$= \frac{1}{\sqrt{2}} \left[\frac{V_m}{R_f + R_L} \right] R_L$$

$$\therefore I_m = \frac{V_m}{R_f + R_L}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \frac{R_L}{R_f + R_L}$$

RMS or AC Output Voltage (V_{rms})

Dividing numerator and denominator by R_L ,

$$V_{rms} = \frac{(V_m/\sqrt{2})}{1 + (R_f/R_L)}$$

Note: If the diode is ideal, then $R_f = 0$. Then $V_{rms} = \frac{V_m}{\sqrt{2}}$ (for an ideal diode).

Rectification Efficiency (η_r)

- Rectification efficiency is defined as the ratio of the dc output power to the ac input power supplied to the rectifier.
- It is given by,

$$\eta_r = \frac{P_{dc}}{P_i} \quad (1)$$

where P_{dc} is the dc output power of the rectifier and P_i is the ac input power to the rectifier.

Rectification Efficiency (η_r)

- The dc output power is given by

$$\begin{aligned} P_{dc} &= I_{dc}^2 R_L \\ &= \left[\frac{2I_m}{\pi} \right]^2 R_L && \because I_{dc} = \frac{2I_m}{\pi} \text{ for a FWR} \\ P_{dc} &= \frac{4I_m^2 R_L}{\pi^2} \end{aligned} \tag{2}$$

Rectification Efficiency (η_r)

- The ac input power is given by

$$P_i = I_{rms}^2 [R_f + R_L]$$
$$= \left[\frac{I_m}{\sqrt{2}} \right]^2 [R_f + R_L]$$

$$\because I_{rms} = \frac{I_m}{\sqrt{2}} \text{ for a HWR}$$

$$P_i = \frac{I_m^2}{2} [R_f + R_L] \quad (3)$$

Rectification Efficiency (η_r)

- Using Eqns. (2) and (3) in (1),

$$\eta_r = \frac{\frac{4I_m^2 R_L}{\pi^2}}{\frac{I_m^2}{2} [R_f + R_L]}$$
$$= \frac{8 R_L}{\pi^2 R_f + R_L}$$

$$\eta_r = \frac{0.8105 R_L}{R_f + R_L}$$

Rectification Efficiency (η_r)

$$\eta_r = \frac{0.8105}{1 + (R_f/R_L)}$$

$$\% \eta_r = \frac{0.8105}{1 + (R_f/R_L)} \times 100\%$$

$$\% \eta_r = \frac{81.05}{1 + (R_f/R_L)} \%$$

Note: Maximum efficiency can be achieved for an ideal diode ($R_f = 0$)

$$\% \eta_{r(max)} = 81.05\%$$

Ripple Factor (γ)

- Ripple factor is the ratio of rms value of ac component present in the rectified output to the dc component of the rectified output.
- It is given by,

$$\gamma = \frac{V_{ac}}{V_{dc}} \quad (1)$$

where V_{ac} is the rms value of ac component present in the rectified output

and V_{dc} is the dc component of the rectified output.

Ripple Factor (γ)

- The total power output is the sum of powers of dc and ac components.

$$P_{total} = P_{dc} + P_{ac}$$

$$\frac{V_{rms}^2}{R_L} = \frac{V_{dc}^2}{R_L} + \frac{V_{ac}^2}{R_L}$$

$$V_{rms}^2 = V_{dc}^2 + V_{ac}^2$$

Ripple Factor (γ)

Dividing throughout by V_{dc}^2 , we get

$$\frac{V_{rms}^2}{V_{dc}^2} = \frac{V_{dc}^2}{V_{dc}^2} + \frac{V_{ac}^2}{V_{dc}^2}$$

$$\left[\frac{V_{rms}}{V_{dc}} \right]^2 = 1 + \left[\frac{V_{ac}}{V_{dc}} \right]^2$$

$$\left[\frac{V_{ac}}{V_{dc}} \right]^2 = \left[\frac{V_{rms}}{V_{dc}} \right]^2 - 1$$

Ripple Factor (γ)

$$\frac{V_{ac}}{V_{dc}} = \sqrt{\left[\frac{V_{rms}}{V_{dc}}\right]^2 - 1}$$

Using Eqn. (1),

$$\boxed{\gamma = \sqrt{\left[\frac{V_{rms}}{V_{dc}}\right]^2 - 1}} \quad (2)$$

Ripple Factor (γ)

In a full-wave rectifier, $V_{rms} = \frac{(V_m/\sqrt{2})}{1+(R_f/R_L)}$ and $V_{dc} = \frac{(2V_m/\pi)}{1+(R_f/R_L)}$

$$\gamma = \sqrt{\left[\frac{\frac{(V_m/\sqrt{2})}{1+(R_f/R_L)}}{\frac{(2V_m/\pi)}{1+(R_f/R_L)}} \right]^2 - 1}$$

$$\gamma = \sqrt{\left(\frac{\pi}{2\sqrt{2}} \right)^2 - 1}$$

Ripple Factor (γ)

$$\gamma = \sqrt{\frac{\pi^2}{8} - 1}$$

$$\gamma = 0.483$$

Ripple Factor (γ)

Note: We have $\gamma = 0.483$.

That means,

$$\gamma = \frac{V_{ac}}{V_{dc}} = 0.483$$

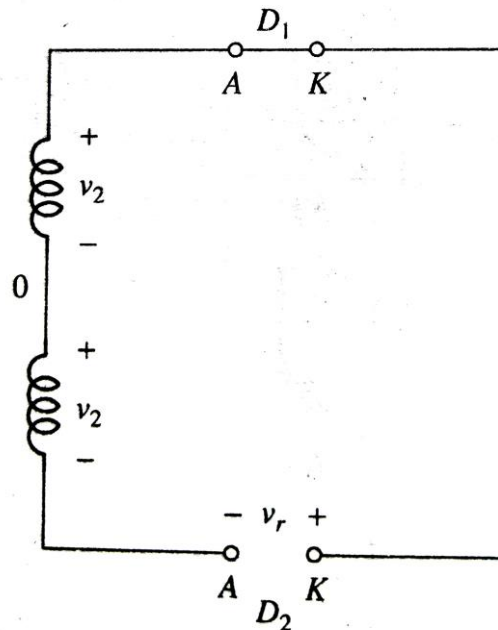
$$V_{ac} = 0.483V_{dc}$$

Peak Inverse Voltage (PIV)

- Peak Inverse Voltage (PIV) is the maximum reverse voltage to which the diode can be subjected.
- If the applied reverse voltage across the diode is greater than its PIV rating, the reverse breakdown of the diode which causes a permanent damage to the diode.

Peak Inverse Voltage (PIV)

- Consider the equivalent circuit of the full-wave rectifier during positive half-cycle.



Conducting diode is replaced by a short circuit and non-conducting diode is replaced by an open circuit.

v_r is the instantaneous reverse voltage across the diode D_2

- From the circuit,

$$v_2 + v_2 = v_r$$

$$v_r = 2v_2$$

$$v_r = 2V_m \sin \omega t$$

- Now,

$$PIV = v_{r(max)}$$

$$PIV = 2V_m$$

Full-Wave Rectifier – Numerical Example 1

The input to the full wave rectifier is $v(t) = 200 \sin 50t$. If R_L is $1 \text{ k}\Omega$ and forward resistance of diode is 50Ω , find:

- i. D.C current through the circuit
- ii. The A.C (rms) value of current through the circuit
- iii. The D.C output voltage
- iv. The A.C power input
- v. The D.C power output
- vi. Rectifier efficiency.

Full-Wave Rectifier – Numerical Example 1

Solution:

$$\text{Given } v(t) = 200 \sin 50t \text{ V}$$

$$R_L = 1 \text{ k}\Omega = 1000\Omega$$

$$R_f = 50 \Omega$$

Comparing $v(t)$ with $V_m \sin \omega t$,

We have $V_m = 200 \text{ V}$ and $\omega = 50 \text{ rad/s}$

Full-Wave Rectifier – Numerical Example 1

$$\begin{aligned}\text{Then, } I_m &= \frac{V_m}{R_f + R_L} \\ &= \frac{200 \text{ V}}{50 \Omega + 1000 \Omega} \\ I_m &= 190.48 \text{ mA}\end{aligned}$$

DC Load Current

$$\begin{aligned}I_{dc} &= \frac{2I_m}{\pi} \\ &= \frac{2 \times 190.48 \text{ mA}}{\pi} \\ I_{dc} &= 121.26 \text{ mA}\end{aligned}$$

RMS Load Current

$$\begin{aligned}I_{rms} &= \frac{I_m}{\sqrt{2}} \\ &= \frac{190.48 \text{ mA}}{\sqrt{2}} \\ I_{rms} &= 134.69 \text{ mA}\end{aligned}$$

DC Output Voltage

$$\begin{aligned}V_{dc} &= I_{dc} R_L \\ &= 121.26 \text{ mA} \times 1 \text{ k}\Omega \\ V_{dc} &= 121.26 \text{ V}\end{aligned}$$

RMS Output Voltage

$$\begin{aligned}V_{rms} &= I_{rms} R_L \\ &= 134.69 \text{ mA} \times 1 \text{ k}\Omega \\ V_{rms} &= 134.69 \text{ V}\end{aligned}$$

Full-Wave Rectifier – Numerical Example 1

AC Input Power

$$\begin{aligned}P_i &= I_{rms}^2 [R_f + R_L] \\ &= (134.69m)^2 \times (50 + 1000) \\ P_i &= 19.048 \text{ W}\end{aligned}$$

DC Output Power

$$\begin{aligned}P_{dc} &= I_{dc}^2 R_L \\ &= (121.26m)^2 \times 1000 \\ P_{dc} &= 14.704 \text{ W}\end{aligned}$$

Rectification Efficiency

$$\begin{aligned}\% \eta_r &= \frac{P_{dc}}{P_i} \times 100 \% \\ &= \frac{14.704}{19.048} \times 100 \% \\ \% \eta_r &= 77.19 \%\end{aligned}$$

Full-Wave Rectifier – Numerical Example 1

Ripple Factor

$$\begin{aligned}\gamma &= \sqrt{\left[\frac{V_{rms}}{V_{dc}}\right]^2 - 1} \\ &= \sqrt{\left(\frac{134.69}{121.26}\right)^2 - 1} \\ \gamma &= 0.483\end{aligned}$$

Peak Inverse Voltage

$$\begin{aligned}PIV &= 2V_m \\ PIV &= 2 \times 200 \text{ V} \\ PIV &= 400 \text{ V}\end{aligned}$$

Full-Wave Rectifier – Numerical Example 2

The input voltage applied to the primary of a 4:1 step down transformer of a full wave centre tap rectifier is 230 V, 50 Hz. If the load resistance is 600 Ω and forward resistance is 20 Ω , determine the following:

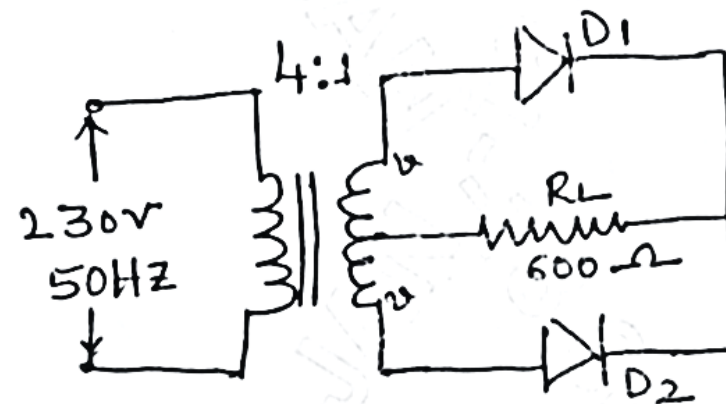
- i. dc output power
- ii. Rectification efficiency
- iii. PIV

Solution:

Given $V_1 = 230\text{ V}$, $f = 50\text{ Hz}$

$N_1 : N_2 = 4 : 1$

$R_L = 600\ \Omega$, $R_f = 20\ \Omega$



Full-Wave Rectifier – Numerical Example 2

For the given circuit,

$$\frac{N_1}{N_2} = \frac{V_1}{2V_2}$$

So
$$2V_2 = \frac{N_2}{N_1} V_1$$

$$V_2 = \frac{N_2}{2N_1} V_1$$

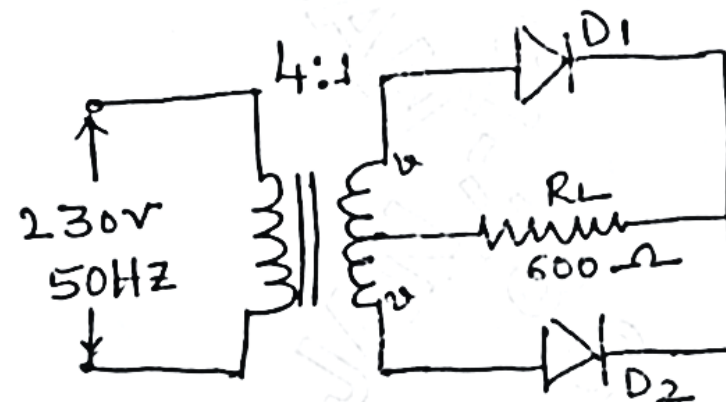
$$V_2 = \frac{1}{2 \times 4} \times 230 V$$

$$V_2 = 28.75 V$$

This value of V_2 is the rms value and the peak value can be found using

$$\begin{aligned} V_m &= \sqrt{2} V_2 \\ &= \sqrt{2} \times 28.75 V \end{aligned}$$

$$V_m = 40.658 V$$



Full-Wave Rectifier – Numerical Example 2

$$\begin{aligned}\text{Then, } I_m &= \frac{V_m}{R_f + R_L} \\ &= \frac{40.658 \text{ V}}{20 \Omega + 600 \Omega} \\ I_m &= 65.577 \text{ mA}\end{aligned}$$

DC Load Current

$$\begin{aligned}I_{dc} &= \frac{2I_m}{\pi} \\ &= \frac{2 \times 65.577 \text{ mA}}{\pi} \\ I_{dc} &= 41.747 \text{ mA}\end{aligned}$$

RMS Load Current

$$\begin{aligned}I_{rms} &= \frac{I_m}{\sqrt{2}} \\ &= \frac{65.577 \text{ mA}}{\sqrt{2}} \\ I_{rms} &= 46.369 \text{ mA}\end{aligned}$$

DC Load Voltage

$$\begin{aligned}V_{dc} &= I_{dc} R_L \\ &= 41.747 \text{ mA} \times 600 \Omega \\ V_{dc} &= 25.048 \text{ V}\end{aligned}$$

RMS Load Voltage

$$\begin{aligned}V_{rms} &= I_{rms} R_L \\ &= 46.369 \text{ mA} \times 600 \Omega \\ V_{rms} &= 27.821 \text{ V}\end{aligned}$$

Full-Wave Rectifier – Numerical Example 2

AC Input Power

$$\begin{aligned}P_i &= I_{rms}^2 [R_f + R_L] \\ &= (46.369m)^2 \times (20 + 600) \\ P_i &= 1.333 \text{ W}\end{aligned}$$

DC Output Power

$$\begin{aligned}P_{dc} &= I_{dc}^2 R_L \\ &= (41.747m)^2 \times 600 \\ P_{dc} &= 1.045 \text{ W}\end{aligned}$$

Rectification Efficiency

$$\begin{aligned}\% \eta_r &= \frac{P_{dc}}{P_i} \times 100 \% \\ &= \frac{1.045}{1.333} \times 100 \% \\ \% \eta_r &= 78.39 \%\end{aligned}$$

Peak Inverse Voltage

$$\begin{aligned}PIV &= 2V_m \\ &= 2 \times 40.658 \text{ V} \\ PIV &= 81.316 \text{ V}\end{aligned}$$

Full-Wave Rectifier – Numerical Example 3

A full wave rectifier uses 2 diodes having internal resistance of $10\ \Omega$ each. The transformer RMS secondary voltage from centre to each end is $200\ V$. Find I_m , I_{dc} , I_{rms} and V_{dc} if the load is $800\ \Omega$.

Solution:

Given $V_2 = 200\ V$, $R_f = 10\ \Omega$ and $R_L = 800\ \Omega$

The given V_2 is the rms value of the input and we can find the peak value using

$$\begin{aligned}V_m &= \sqrt{2}V_2 \\ &= \sqrt{2} \times 200\ V\end{aligned}$$

$$V_m = 282.84\ V$$

Full-Wave Rectifier – Numerical Example 3

$$\begin{aligned}\text{Then, } I_m &= \frac{V_m}{R_f + R_L} \\ &= \frac{282.84 \text{ V}}{10 \Omega + 800 \Omega} \\ I_m &= 349.185 \text{ mA}\end{aligned}$$

DC Load Current

$$\begin{aligned}I_{dc} &= \frac{2I_m}{\pi} \\ &= \frac{2 \times 349.185 \text{ mA}}{\pi} \\ I_{dc} &= 222.298 \text{ mA}\end{aligned}$$

RMS Load Current

$$\begin{aligned}I_{rms} &= \frac{I_m}{\sqrt{2}} \\ &= \frac{349.185 \text{ mA}}{\sqrt{2}} \\ I_{rms} &= 246.911 \text{ mA}\end{aligned}$$

DC Load Voltage

$$\begin{aligned}V_{dc} &= I_{dc} R_L \\ &= 222.298 \text{ mA} \times 800 \Omega \\ V_{dc} &= 177.836 \text{ V}\end{aligned}$$

RMS Load Voltage

$$\begin{aligned}V_{rms} &= I_{rms} R_L \\ &= 246.911 \text{ mA} \times 800 \Omega \\ V_{rms} &= 148.146 \text{ V}\end{aligned}$$

Full-Wave Rectifier – Numerical Example 4

A full wave rectifier supplies a load of 1000Ω . The ac voltage applied to it is $200 - 0 - 200 V$ (*rms*). If diode resistance is neglected, calculate (i) I_{dc} (ii) I_{rms} (iii) efficiency (η).

Solution:

Given $V_2 = 200 V$, $R_L = 1000 \Omega$. Take $R_f = 0 \Omega$ (given R_f is neglected)

The given V_2 is the rms value of the input and we can find the peak value using

$$\begin{aligned} V_m &= \sqrt{2}V_2 \\ &= \sqrt{2} \times 200 V \end{aligned}$$

$$V_m = 282.84 V$$

Full-Wave Rectifier – Numerical Example 4

$$\begin{aligned}\text{Then, } I_m &= \frac{V_m}{R_f + R_L} \\ &= \frac{282.84 \text{ V}}{0 \Omega + 1000 \Omega} \\ I_m &= 282.84 \text{ mA}\end{aligned}$$

DC Load Current

$$\begin{aligned}I_{dc} &= \frac{2I_m}{\pi} \\ &= \frac{2 \times 282.84 \text{ mA}}{\pi} \\ I_{dc} &= 180.06 \text{ mA}\end{aligned}$$

RMS Load Current

$$\begin{aligned}I_{rms} &= \frac{I_m}{\sqrt{2}} \\ &= \frac{282.84 \text{ mA}}{\sqrt{2}} \\ I_{rms} &= 200 \text{ mA}\end{aligned}$$

Full-Wave Rectifier – Numerical Example 4

AC Input Power

$$\begin{aligned}P_i &= I_{rms}^2 [R_f + R_L] \\ &= (200m)^2 \times (0 + 1000)\end{aligned}$$

$$P_i = 40 \text{ W}$$

DC Output Power

$$\begin{aligned}P_{dc} &= I_{dc}^2 R_L \\ &= (180.06 \text{ m})^2 \times 1000\end{aligned}$$

$$P_{dc} = 32.42 \text{ W}$$

Rectification Efficiency

$$\begin{aligned}\% \eta_r &= \frac{P_{dc}}{P_i} \times 100 \% \\ &= \frac{32.42}{40} \times 100 \%\end{aligned}$$

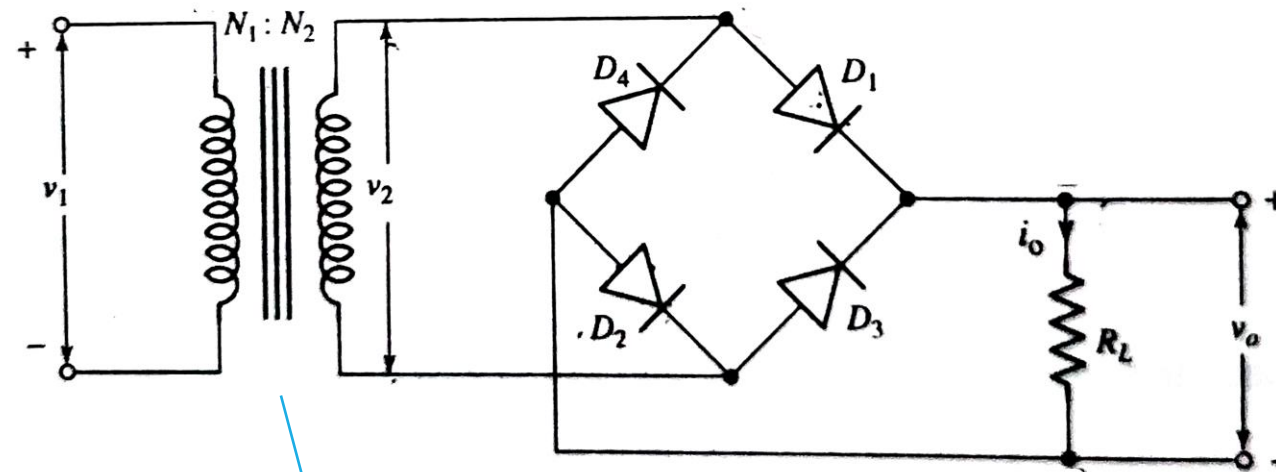
$$\% \eta_r = 81.05 \%$$

Bridge Rectifier

- A *Bridge Rectifier (BR)* is a full-wave rectifier constructed using four diodes connected in the form of a bridge.
 - It converts both the half-cycles of the input ac to pulsating dc.

Bridge Rectifier

Circuit Diagram



Step-down
Transformer

Bridge Rectifier

- The bridge rectifier consists of four diodes D_1 , D_2 , D_3 and D_4 which are arranged in the form of a bridge as shown.
- A step-down transformer is used to reduce the available ac voltage to the required level.
- Resistor R_L is the load resistance which consumes power from the rectifier.

Bridge Rectifier

Operation

- Consider the single phase ac input signal given by

$$v_1 = V_m \sin \omega t \quad (1)$$

- We have transformer turns ratio

$$\frac{N_1}{N_2} = \frac{v_1}{v_2}$$

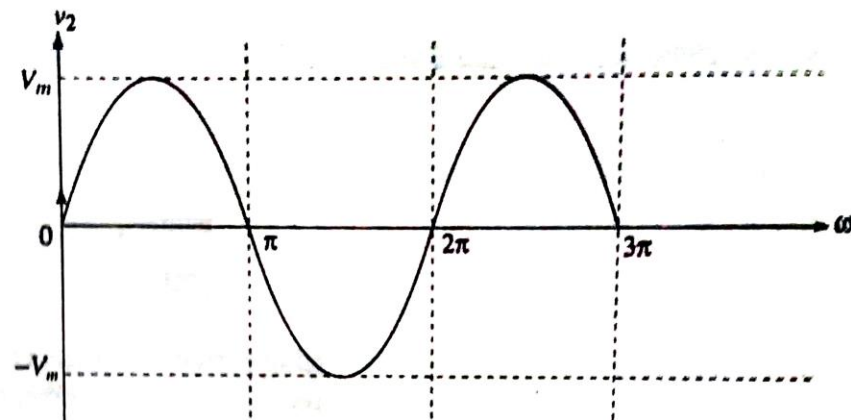
- Rearranging, we can write transformer secondary voltage as

$$v_2 = \frac{N_2}{N_1} V_m \sin \omega t \quad (2)$$

Bridge Rectifier

- For simplicity, consider $N_1 = N_2$.
- Then, Eqn. (2) can be written as

$$v_2 = V_m \sin \omega t \quad (3)$$



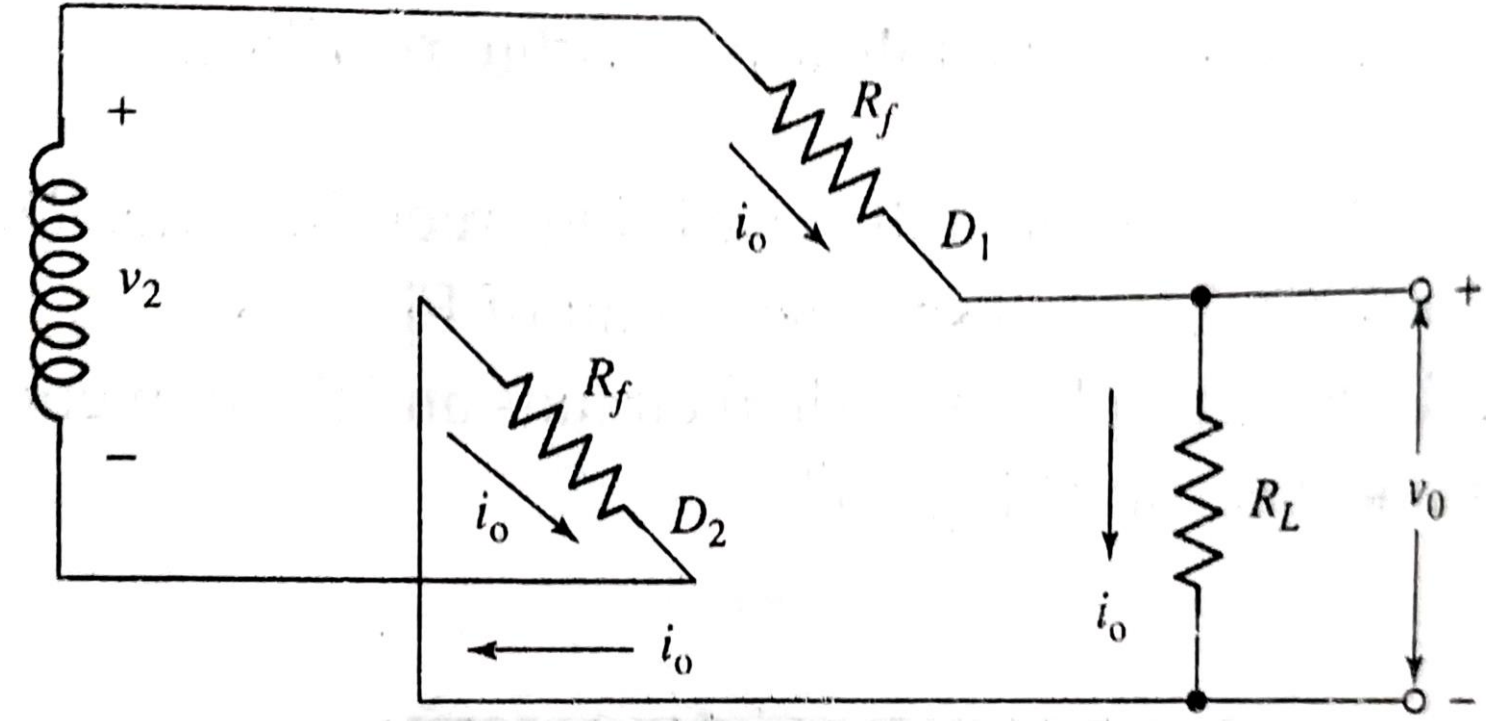
Bridge Rectifier

(i) During positive half-cycle of ac supply ($0 \leq \omega t \leq \pi$)

- The diodes D_1 and D_2 are forward biased and hence they conduct, whereas the diodes D_3 and D_4 are reverse biased and hence they remain *off*.
- The conducting diodes can be replaced by forward resistance R_f and the non-conducting diodes can be replaced by open circuit and the equivalent circuit can be drawn as shown.

Bridge Rectifier

Equivalent circuit during positive half-cycle



Bridge Rectifier

- From the circuit,

$$i_o = \frac{v_2}{R_f + R_L + R_f}$$

- Using $v_2 = V_m \sin \omega t$, (from Eqn. (3))

$$i_o = \frac{V_m \sin \omega t}{2R_f + R_L}$$

- We can write

$$i_o = I_m \sin \omega t \quad (4)$$

where $I_m = \frac{V_m}{2R_f + R_L}$ is the peak value of load current.

- The output voltage is then $v_o = i_o R_L$

Bridge Rectifier

(ii) During negative half-cycle of ac supply ($\pi \leq \omega t \leq 2\pi$)

- The diodes D_3 and D_4 are forward biased and hence they conduct, whereas the diodes D_1 and D_2 are reverse biased and hence they remain *off*.
- The conducting diodes can be replaced by forward resistance R_f and the non-conducting diodes can be replaced by open circuit.

Bridge Rectifier

- As the equivalent circuit is similar, we can write

$$i_o = I_m \sin \omega t \quad (5)$$

where $I_m = \frac{V_m}{2R_f + R_L}$ is the peak value of load current.

- The output voltage is then $v_o = i_o R_L$

Bridge Rectifier

- Using Eqns. (4) and (5), we can write

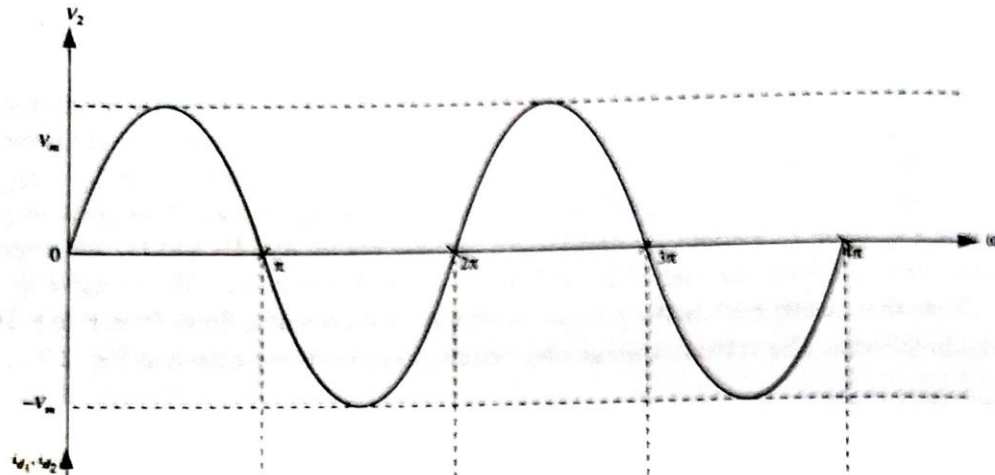
$$i_0 = I_m \sin \omega t \quad (6)$$

where $I_m = \frac{V_m}{2R_f + R_L}$ is the peak value of load current.

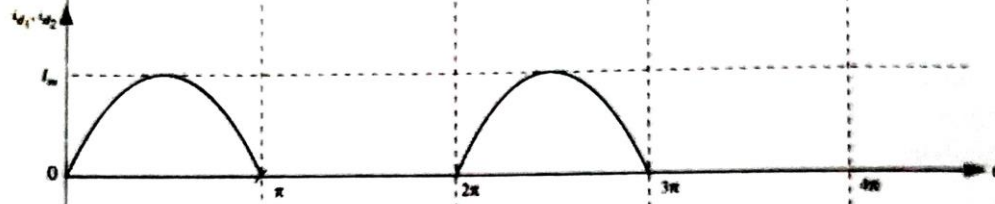
Note: If the diodes are ideal, then $R_f = 0$. Then $I_m = \frac{V_m}{R_L}$ (for an ideal diode).

Waveforms

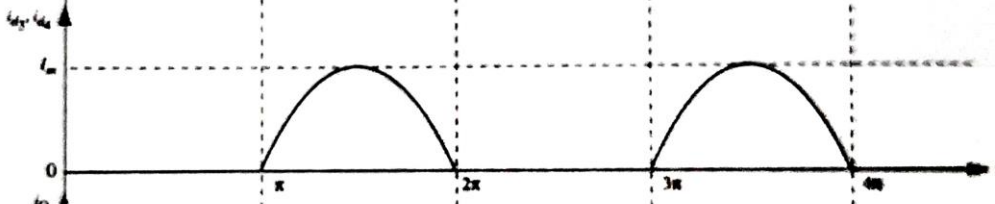
Input voltage



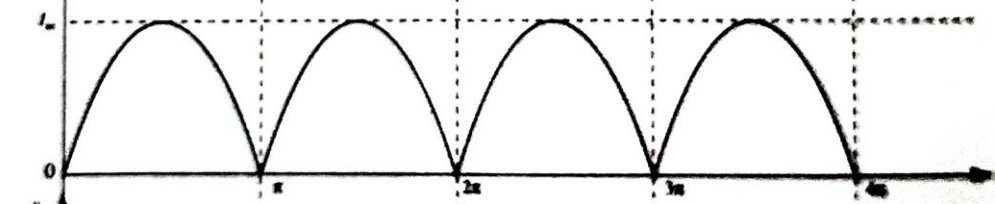
Current through diode D_1 & D_2



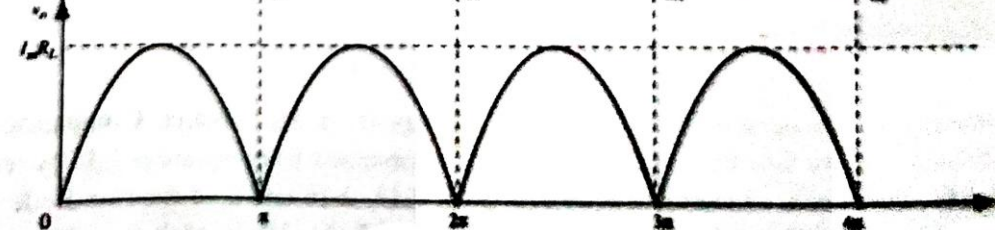
Current through diode D_3 & D_4



Load current



Output voltage



Bridge Rectifier

Average or DC Load Current (I_{dc})

$$I_{dc} = \frac{2I_m}{\pi}$$

Average or DC Output Voltage (V_{dc})

$$V_{dc} = \frac{2V_m}{\pi} \frac{R_L}{2R_f + R_L}$$

$$V_{dc} = \frac{(2V_m/\pi)}{1 + (2R_f/R_L)}$$

Note: If the diodes are ideal, then $R_f = 0$. Then $V_{dc} = \frac{2V_m}{\pi}$ (for an ideal diode).

Bridge Rectifier

RMS or AC Load Current (I_{rms})

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

RMS or AC Output Voltage (V_{rms})

$$V_{rms} = \frac{V_m}{\sqrt{2}} \frac{R_L}{2R_f + R_L}$$
$$V_{rms} = \frac{(V_m/\sqrt{2})}{1 + (2R_f/R_L)}$$

Note: If the diodes are ideal, then $R_f = 0$. Then $V_{rms} = \frac{V_m}{\sqrt{2}}$ (for an ideal diode).

Bridge Rectifier

Rectification Efficiency (η_r)

$$\% \eta_r = \frac{81.05}{1 + (2R_f/R_L)} \%$$

Note: If the diodes are ideal, then $R_f = 0$. Then $\% \eta_{r(max)} = 81.05\%$

Ripple Factor

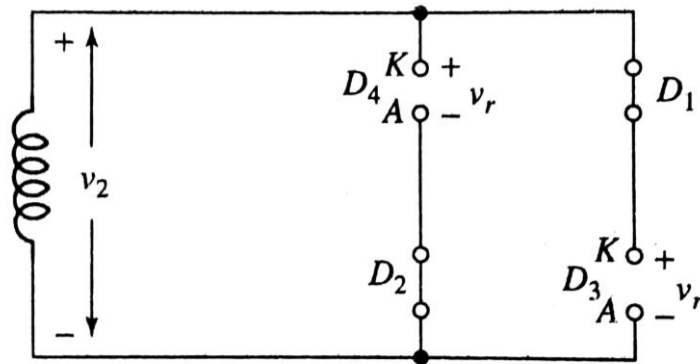
$$\gamma = \sqrt{\left[\frac{V_{rms}}{V_{dc}} \right]^2 - 1}$$
$$\gamma = 0.483$$

Bridge Rectifier

Peak Inverse Voltage (PIV)

Consider the equivalent circuit of the full-wave bridge rectifier during positive half-cycle.

Conducting diode is replaced by a short circuit and non-conducting diode is replaced by an open circuit.



- From the circuit,

$$v_r = v_2$$

$$v_r = V_m \sin \omega t$$

- Now,

$$PIV = v_{r(max)}$$

$$PIV = V_m$$

v_r is the instantaneous reverse voltage across the diode D_2

Bridge Rectifier – Numerical Example 1

Determine the peak output voltage and current for a bridge rectifier circuit when the secondary RMS voltage is 30 V , load resistance is $300\ \Omega$ and the diode forward drop is 0.7 V .

Solution:

Given $V_i = 30\text{ V}$, $R_L = 300\ \Omega$, $V_F = 0.7\text{ V}$

The given V_i is the rms value and the peak value can be found using

$$\begin{aligned}V_{pi} &= \sqrt{2}V_i \\ &= \sqrt{2} \times 30\text{ V} \\ V_{pi} &= 42.42\text{ V}\end{aligned}$$

Bridge Rectifier – Numerical Example 1

i) The peak output voltage

$$\begin{aligned}V_{po} &= V_{pi} - 2V_F \\ &= 42.42 \text{ V} - (2 \times 0.7 \text{ V}) \\ V_{po} &= 41.02 \text{ V}\end{aligned}$$

ii) The peak current

$$\begin{aligned}I_p &= \frac{V_{po}}{R_L} \\ &= \frac{41.02 \text{ V}}{300 \Omega} \\ I_p &= 136.73 \text{ mA}\end{aligned}$$

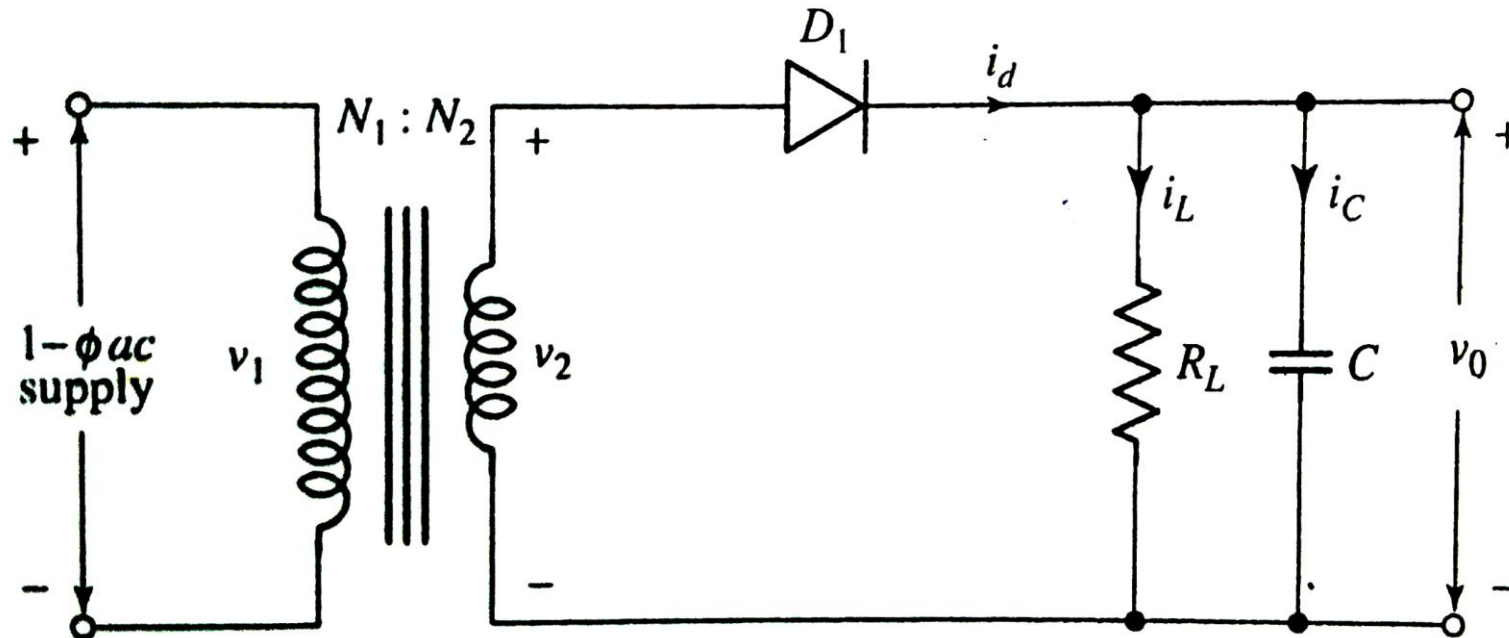
Capacitor Filter Circuit

Capacitor Filter Circuit

- Filter is a circuit used to reduce the ripple content present in the rectified output.
- A capacitor connected in parallel with the load resistor acts as a filter by reducing the ripple content present in the rectified output.
- The output from half-wave and full-wave rectifiers is not a smooth dc due to the ripple content.
 - The ripple content of half-wave rectified output is 121% of dc component whereas it is 48.3% of dc component in full-wave rectified output.
- In order to obtain smooth dc, it is necessary to filter out the ripple content.

Half-Wave Rectifier with a 'C' Filter

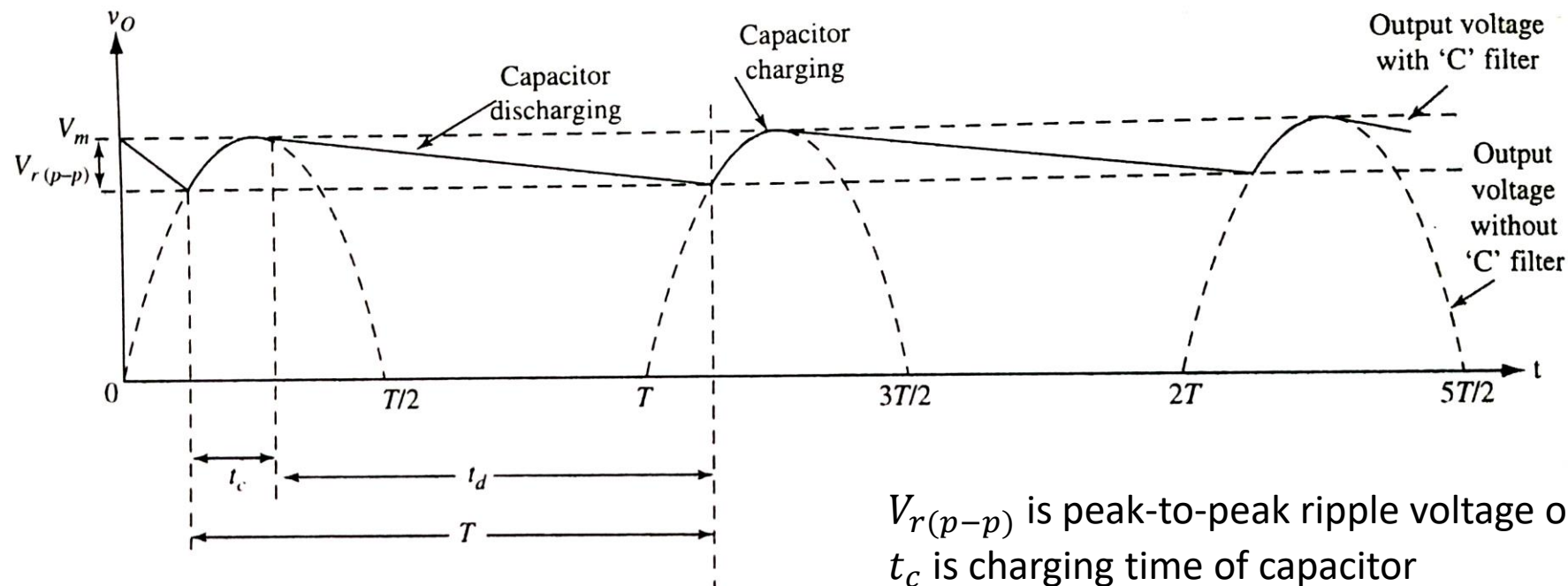
Circuit Diagram



Half-wave rectifier with capacitor filter

Half-Wave Rectifier with a 'C' Filter

Output Voltage Waveform



Output voltage waveforms with and without 'C' filter

$V_{r(p-p)}$ is peak-to-peak ripple voltage on capacitor

t_c is charging time of capacitor

t_d is discharging time of capacitor

T is the time period of output waveform ($T = t_c + t_d$)

Half-Wave Rectifier with a 'C' Filter

- During the positive half-cycle of the ac supply, the diode conducts and charges the capacitor to the peak value V_m of the input voltage.
- When the input voltage falls below V_m , the diode stops conducting.
- Now, the capacitor starts discharging into the load R_L and the voltage on capacitor decreases.
- The discharging of the capacitor continues till the diode starts conducting again and charges the capacitor in the next positive half-cycle of the ac supply.

Half-Wave Rectifier with a 'C' Filter

- From the waveforms, we find that without capacitor filter, the output voltage v_o varies between 0 and V_m and with capacitor filter, the variation is between $[V_m - V_{r(p-p)}]$ and V_m .
- It can be observed that, with filter, the variation in v_o is smaller than that without filter.
- Thus, by using a capacitor filter, the ripple content of the output voltage is considerably reduced.

Half-Wave Rectifier with a 'C' Filter

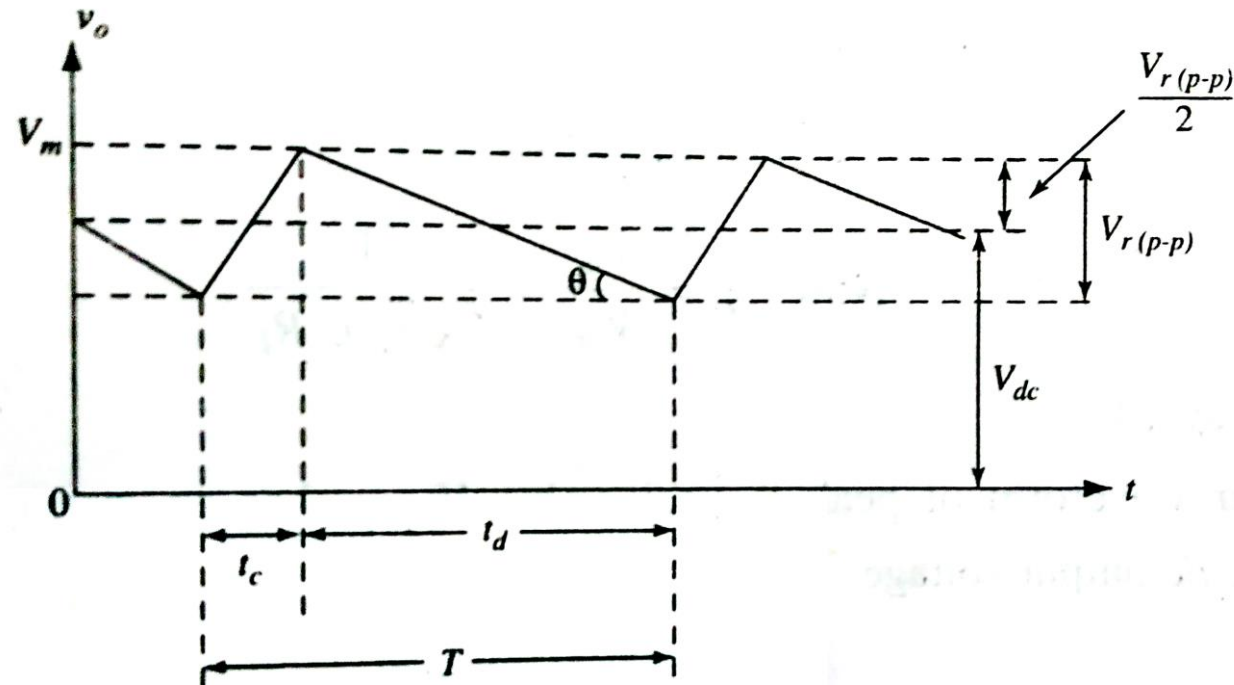
Expression for Ripple Factor

- We know that, the ripple factor

$$\gamma = \frac{V_{ac}}{V_{dc}} \quad (1)$$

- For large value of capacitance, the ripple voltage can be approximated to a triangular waveform with a peak-to-peak value $V_{r(p-p)}$.

Half-Wave Rectifier with a 'C' Filter



Ripple voltage on capacitor approximated by triangular waveform

Half-Wave Rectifier with a 'C' Filter

- For a triangular waveform with peak-to-peak value of $V_{r(p-p)}$, the rms value is given by

$$V_{ac} = \frac{V_{r(p-p)}}{2\sqrt{3}} \quad (2) \quad \because V_{rms} = \frac{V_m}{\sqrt{3}} \text{ for a triangular wave}$$

- During the period t_d , the capacitor discharges a steady current into the load, i.e.,

$$i_c = I_{dc} = C \frac{dv_o}{dt} \quad (3)$$

Half-Wave Rectifier with a 'C' Filter

- From the figure, during discharge,

$$\frac{dv_o}{dt} = \frac{V_{r(p-p)}}{t_d} \quad (4)$$

- Using Eqn. (4) in (3),

$$I_{dc} = C \frac{V_{r(p-p)}}{t_d}$$
$$V_{r(p-p)} = \frac{I_{dc} t_d}{C} \quad (5)$$

Half-Wave Rectifier with a 'C' Filter

- We know that $T = t_c + t_d$.
- When C is large, $t_c \ll t_d$. So $t_d \cong T$.
- So from Eqn. (5),

$$V_{r(p-p)} = \frac{I_{dc}T}{C}$$

- Using $T = 1/f$

$$\boxed{V_{r(p-p)} = \frac{I_{dc}}{fC}} \quad (6)$$

Half-Wave Rectifier with a 'C' Filter

$$V_{r(p-p)} = \frac{V_{dc}}{fCR_L} \quad (7) \quad \therefore I_{dc} = \frac{V_{dc}}{R_L}$$

- From Eqn. (2), $V_{r(p-p)} = 2\sqrt{3}V_{ac}$
- Using this in (7), we get

$$2\sqrt{3}V_{ac} = \frac{V_{dc}}{fCR_L}$$
$$\frac{V_{ac}}{V_{dc}} = \frac{1}{2\sqrt{3}fCR_L}$$

Half-Wave Rectifier with a 'C' Filter

- Hence the ripple factor

$$\gamma = \frac{1}{2\sqrt{3}fCR_L} \quad (8)$$

Half-Wave Rectifier with a 'C' Filter

DC Output Voltage (V_{dc})

- The triangular waveform of peak-to-peak value $V_{r(p-p)}$ has an average value of $\frac{V_{r(p-p)}}{2}$.
- Therefore, the dc output voltage

$$V_{dc} = V_m - \frac{V_{r(p-p)}}{2} \quad (9)$$

- Substituting for $V_{r(p-p)}$ from Eqn. (6),

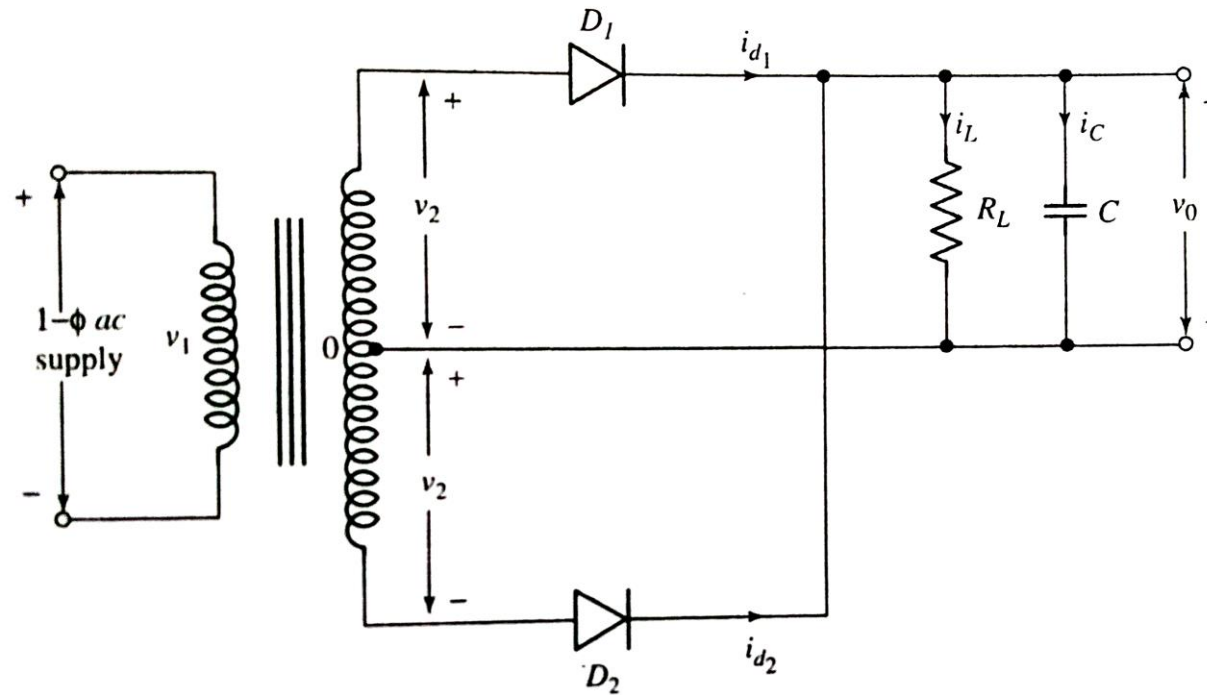
$$V_{dc} = V_m - \frac{I_{dc}}{2fC} \quad (10)$$

Half-Wave Rectifier with a 'C' Filter

- If the capacitance C is large, $\frac{1}{2fC}$ is small.
- Hence, $V_{dc} \cong V_m$ (for large C)

Full-Wave Rectifier with a 'C' Filter

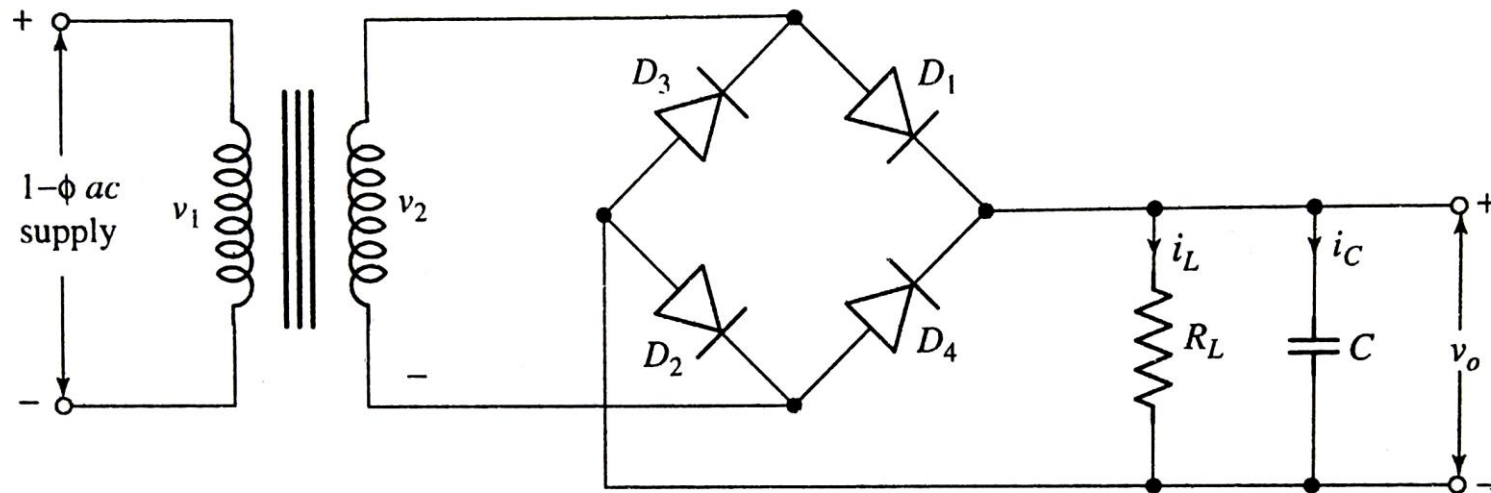
Circuit Diagram



Full-wave rectifier with capacitor filter

Full-Wave Rectifier with a 'C' Filter

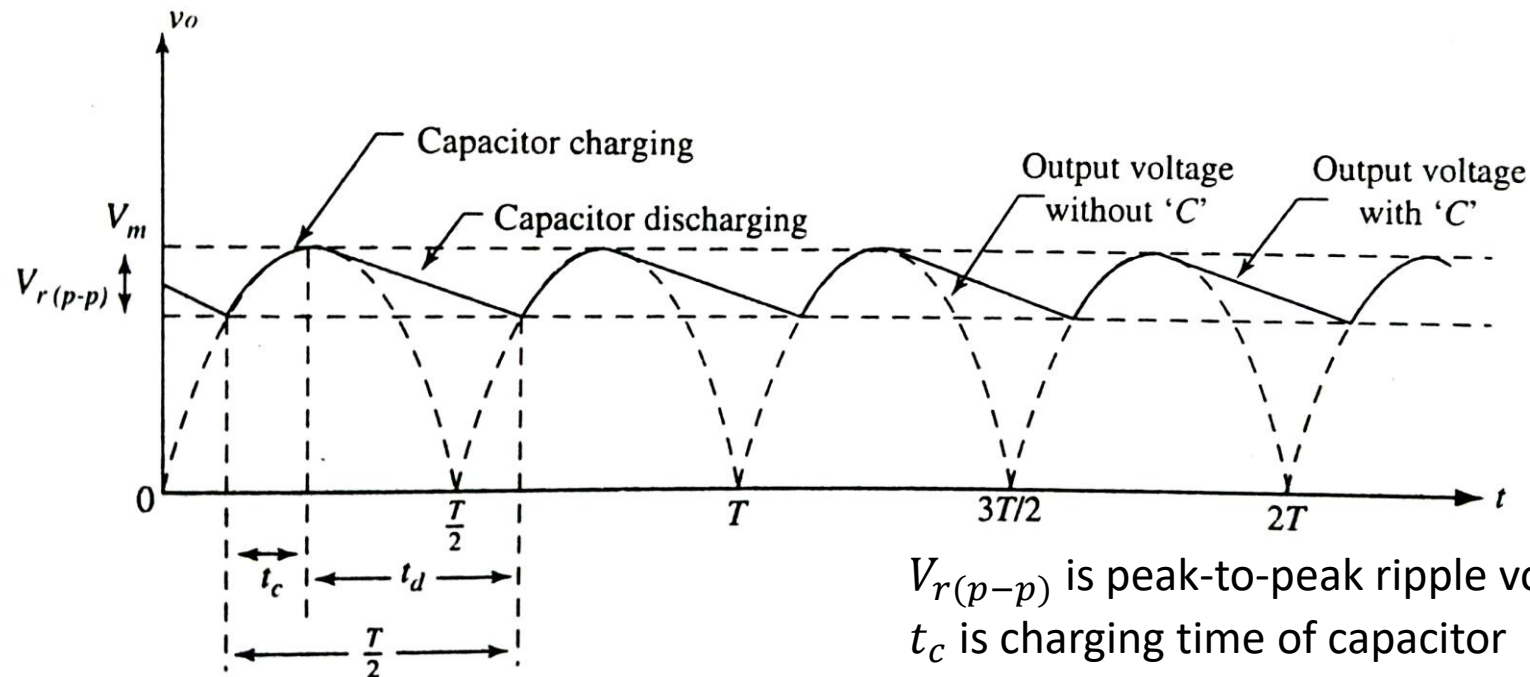
Circuit Diagram



Bridge rectifier with capacitor filter

Full-Wave Rectifier with a 'C' Filter

Output Voltage Waveform



Output voltage waveforms with and without 'C' filter

$V_{r(p-p)}$ is peak-to-peak ripple voltage on capacitor
 t_c is charging time of capacitor
 t_d is discharging time of capacitor
 $\frac{T}{2}$ is the time period of output waveform $\left(\frac{T}{2} = t_c + t_d\right)$

Full-Wave Rectifier with a 'C' Filter

For Two-Diode Full-Wave Rectifier with 'C' Filter

- During the positive half-cycle of the ac supply, the diode D_1 conducts and charges the capacitor to the peak value V_m of the input voltage.
- When the input voltage falls below V_m , the diode stops conducting.
- Now, the capacitor starts discharging into the load R_L and the voltage on capacitor decreases.
- The discharging of the capacitor continues until the diode D_2 starts conducting in the next half-cycle and charges the capacitor again.

Full-Wave Rectifier with a 'C' Filter

For Full-Wave Bridge Rectifier with 'C' Filter

- During the positive half-cycle of the ac supply, the diodes D_1 and D_2 conduct and charge the capacitor to the peak value V_m of the input voltage.
- When the input voltage falls below V_m , the diodes D_1 and D_2 stop conducting.
- Now, the capacitor starts discharging into the load R_L and the voltage on capacitor decreases.
- The discharging of the capacitor continues until the diodes D_3 and diodes D_1 and D_4 start conducting in the next half-cycle and charge the capacitor again.

Full-Wave Rectifier with a 'C' Filter

- From the waveforms, we find that without capacitor filter, the output voltage v_o varies between 0 and V_m and with capacitor filter, the variation is between $[V_m - V_{r(p-p)}]$ and V_m .
- It can be observed that, with filter, the variation in v_o is smaller than that without filter.
- Thus, by using a capacitor filter, the ripple content of the output voltage is considerably reduced.

Full-Wave Rectifier with a 'C' Filter

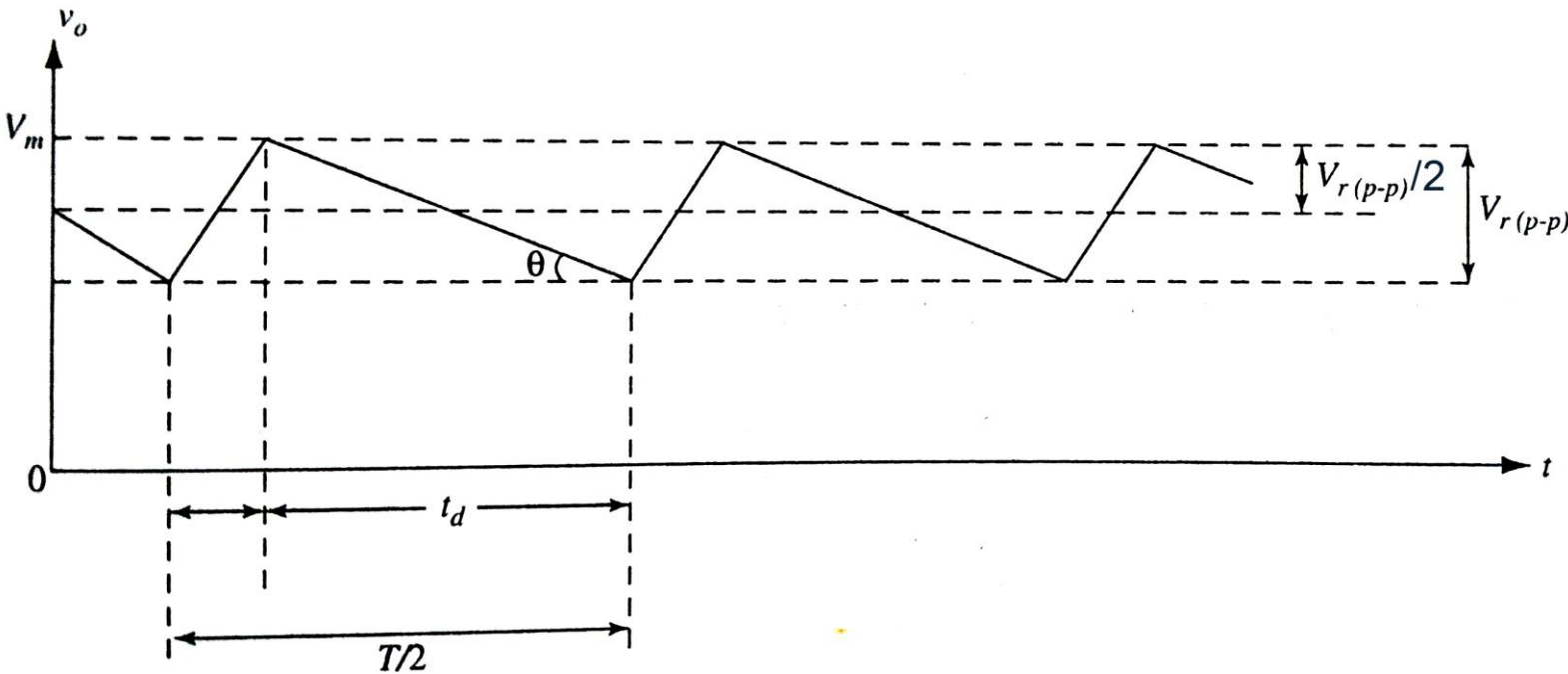
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- For large value of capacitance, the ripple voltage can be approximated to a triangular waveform with a peak-to-peak value $V_{r(p-p)}$.

Full-Wave Rectifier with a 'C' Filter



Ripple voltage on capacitor approximated by triangular waveform

Full-Wave Rectifier with a 'C' Filter

- For a triangular waveform with peak-to-peak value of $V_{r(p-p)}$, the rms value is given by

$$V_{ac} = \frac{V_{r(p-p)}}{2\sqrt{3}} \quad (2) \quad \because V_{rms} = \frac{V_m}{\sqrt{3}} \text{ for a triangular wave}$$

- During the period t_d , the capacitor discharges a steady current into the load, i.e.,

$$i_c = I_{dc} = C \frac{dv_o}{dt} \quad (3)$$

Full-Wave Rectifier with a 'C' Filter

- From the figure, during discharge,

$$\frac{dv_o}{dt} = \frac{V_{r(p-p)}}{t_d} \quad (4)$$

- Using Eqn. (4) in (3),

$$I_{dc} = C \frac{V_{r(p-p)}}{t_d}$$
$$V_{r(p-p)} = \frac{I_{dc} t_d}{C} \quad (5)$$

Full-Wave Rectifier with a 'C' Filter

- We know that $\frac{T}{2} = t_c + t_d$.
- When C is large, $t_c \ll t_d$. So $t_d \cong \frac{T}{2}$.
- So from Eqn. (5),

$$V_{r(p-p)} = \frac{I_{dc}T}{2C}$$

- Using $T = 1/f$

$$\boxed{V_{r(p-p)} = \frac{I_{dc}}{2fC}} \quad (6)$$

Full-Wave Rectifier with a 'C' Filter

$$V_{r(p-p)} = \frac{V_{dc}}{2fCR_L} \quad (7) \quad \therefore I_{dc} = \frac{V_{dc}}{R_L}$$

- From Eqn. (2), $V_{r(p-p)} = 2\sqrt{3}V_{ac}$
- Using this in (7), we get

$$2\sqrt{3}V_{ac} = \frac{V_{dc}}{2fCR_L}$$
$$\frac{V_{ac}}{V_{dc}} = \frac{1}{4\sqrt{3}fCR_L}$$

Full-Wave Rectifier with a 'C' Filter

- Hence the ripple factor

$$\gamma = \frac{1}{4\sqrt{3}fCR_L} \quad (8)$$

Full-Wave Rectifier with a 'C' Filter

DC Output Voltage (V_{dc})

- The triangular waveform of peak-to-peak value $V_{r(p-p)}$ has an average value of $\frac{V_{r(p-p)}}{2}$.
- Therefore, the dc output voltage

$$V_{dc} = V_m - \frac{V_{r(p-p)}}{2} \quad (9)$$

- Substituting for $V_{r(p-p)}$ from Eqn. (6),

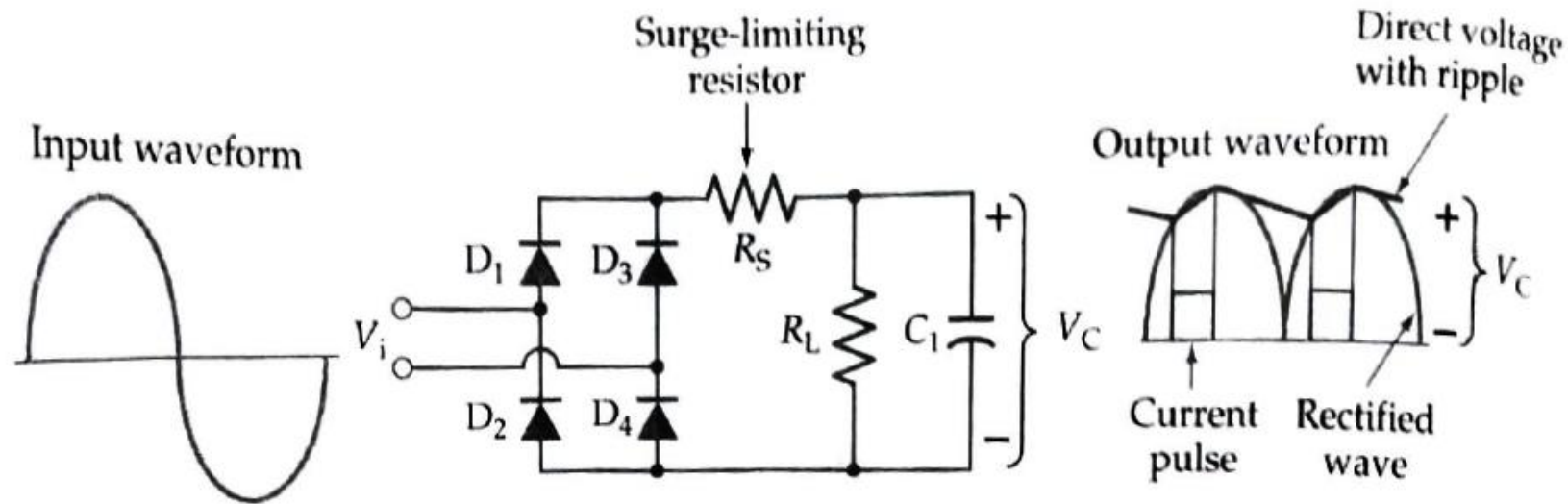
$$V_{dc} = V_m - \frac{I_{dc}}{4fC} \quad (10)$$

Full-Wave Rectifier with a 'C' Filter

- If the capacitance C is large, $\frac{1}{4fC}$ is small.
- Hence, $V_{dc} \cong V_m$ (for large C)

Full-Wave Rectifier Power Supply

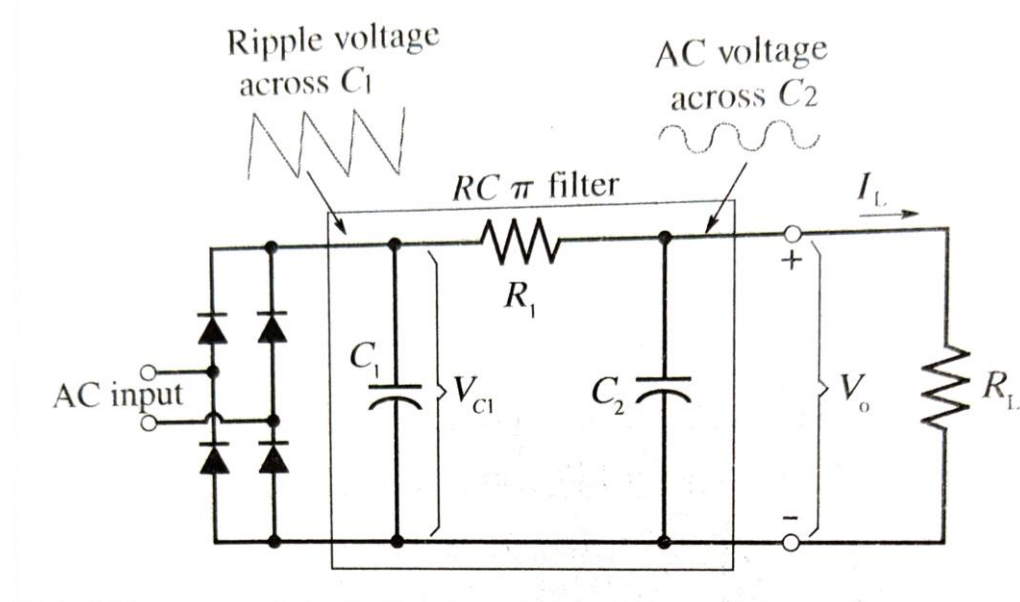
- Full-wave rectifiers require filter circuits in order to convert the output waveform to direct voltage.



Bridge rectifier circuit with a reservoir capacitor to smooth the output voltage and a surge-limiting resistor to protect the diodes.

RC π Filter

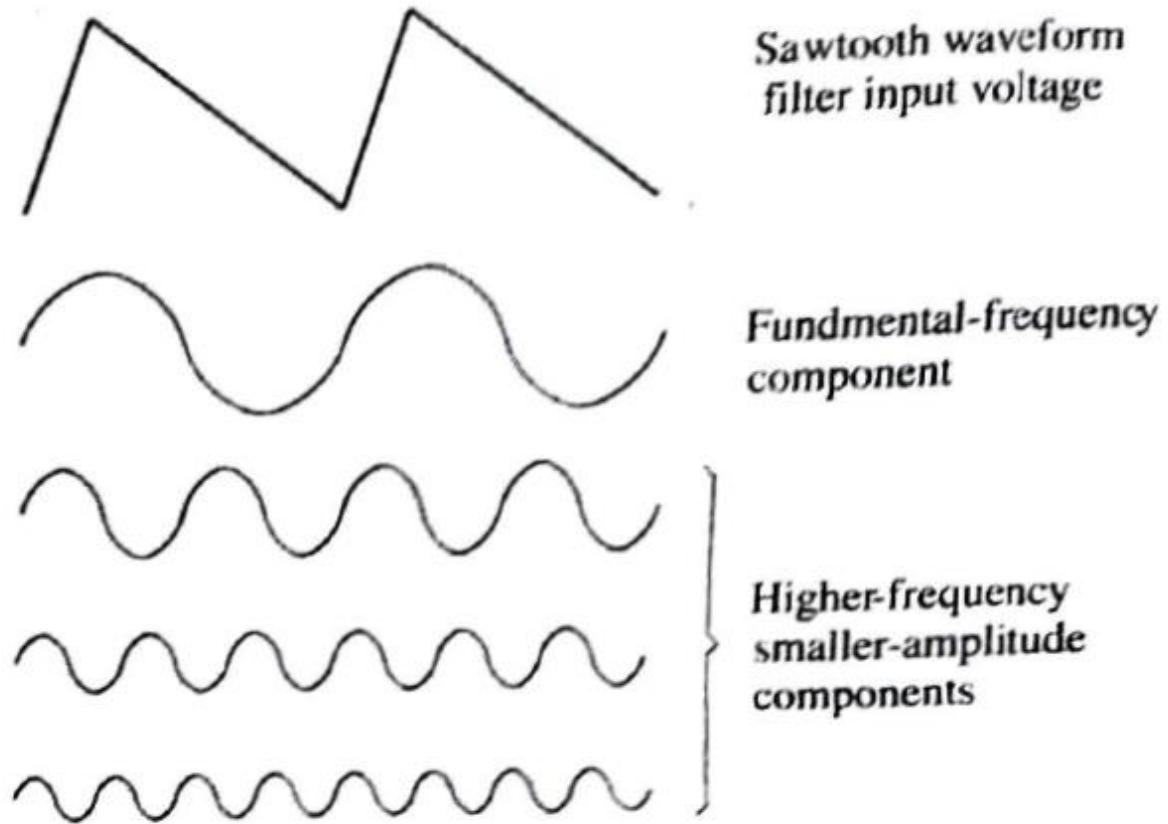
- The ripple voltage that appears across the reservoir capacitor in a rectifier power supply can be attenuated using an additional resistor and capacitor, which together function as an ac voltage divider.



RC π Filter

- C_1 is the reservoir capacitor, and R_1 and C_2 are the additional components.
- The combination of C_1 , R_1 and C_2 is referred to as a π filter, because of the π -shaped arrangement of the circuit components.

RC π Filter



RC π Filter

- The reservoir capacitor produces a sawtooth (ripple) waveform across C_1 regardless of the presence of the additional components.
- The sawtooth waveform is composed of a fundamental ac voltage (same frequency as the ripple) and a number of smaller-amplitude, higher-frequency harmonic components.
- Due to their higher frequencies, the harmonic components are more severely attenuated than the fundamental frequency component by the voltage division across R_1 and C_2 .
- The waveform developed across C_2 (the filter output) is essentially an attenuated version of the sinusoidal fundamental component.

RC π Filter

- The peak value of the fundamental component of the sawtooth waveform is

$$v_p = \frac{V_r}{\pi}$$

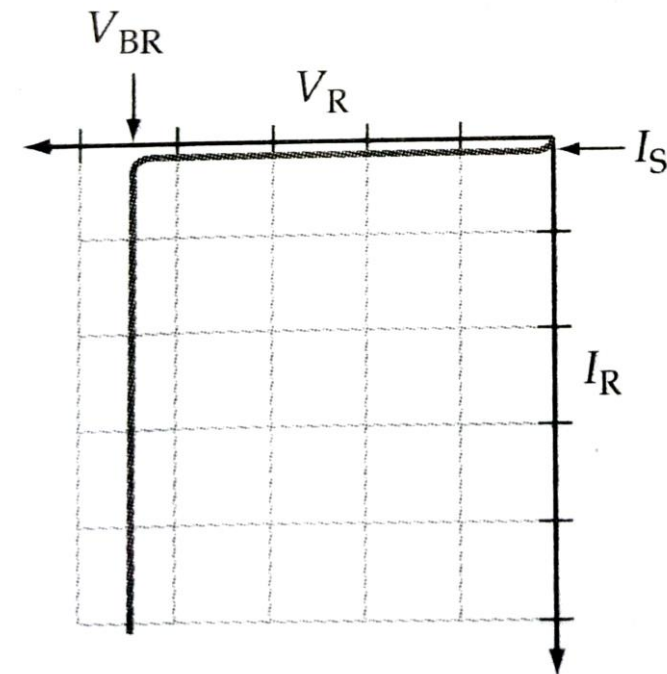
- where V_r is the ripple voltage peak-to-peak amplitude
- The ac voltage developed across C_2 is the filter ac output and is given by

$$v_o = \frac{v_i X_{C_2}}{\sqrt{R_1^2 + X_{C_2}^2}}$$

Zener Diode

Junction Breakdown

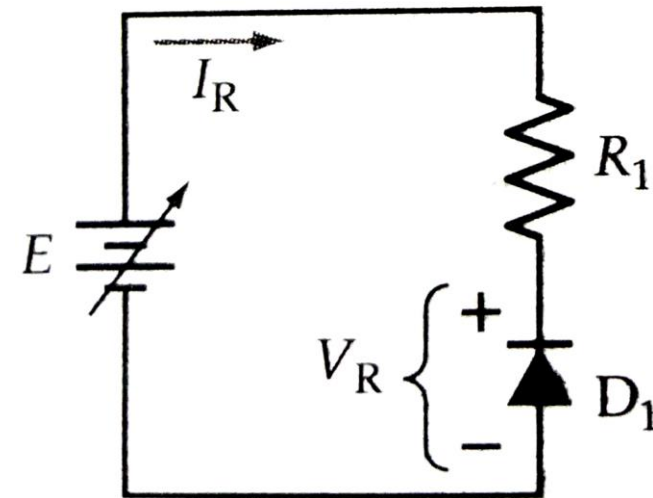
- When a PN junction diode is reverse biased, there is normally only a very small reverse saturation current I_S .
- When the reverse voltage is sufficiently increased, the junction breaks down and a large reverse current flows.



Diode reverse characteristic

Junction Breakdown

- If the reverse current is limited by means of a suitable series-connected resistor R_1 , the power dissipation in the diode can be kept to a level that will not destroy the device.
- In this case, the diode may be operated continuously in reverse breakdown.
- The reverse current returns to its normal level when the voltage is reduced below the reverse breakdown level.



Diode-resistor circuit

Junction Breakdown

- There are two mechanisms that cause breakdown in a reverse biased PN junction:
 - Zener breakdown
 - Avalanche breakdown

Junction Breakdown

Zener Breakdown

- When the depletion region is very narrow, the electric field strength produced by a reverse bias voltage can be very high.
- The high-intensity electric field causes electrons to break away from their atoms, thus converting the depletion region from an insulating material into a conductor.
- This is called *ionization by electric field*, also called *Zener breakdown*.
- This usually occurs with reverse bias voltage less than 5 V.

Junction Breakdown

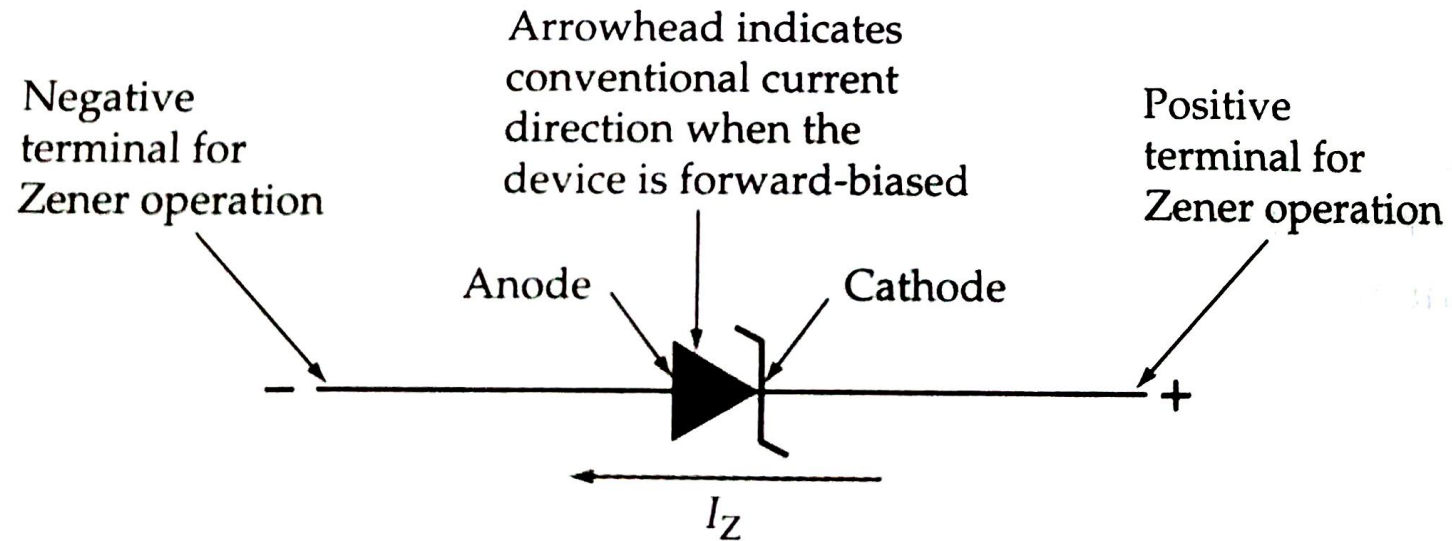
Avalanche Breakdown

- When the depletion region is too wide, the electrons in the reverse saturation current can be given sufficient energy to strike atoms within the depletion region and cause other electrons to break free.
- The electrons released in this way collide with other atoms to produce more free electrons in an avalanche effect.
- This is called *ionization by collision*, also called *avalanche breakdown*.
- This usually occurs with reverse bias voltage above 5 V.

Zener Diode

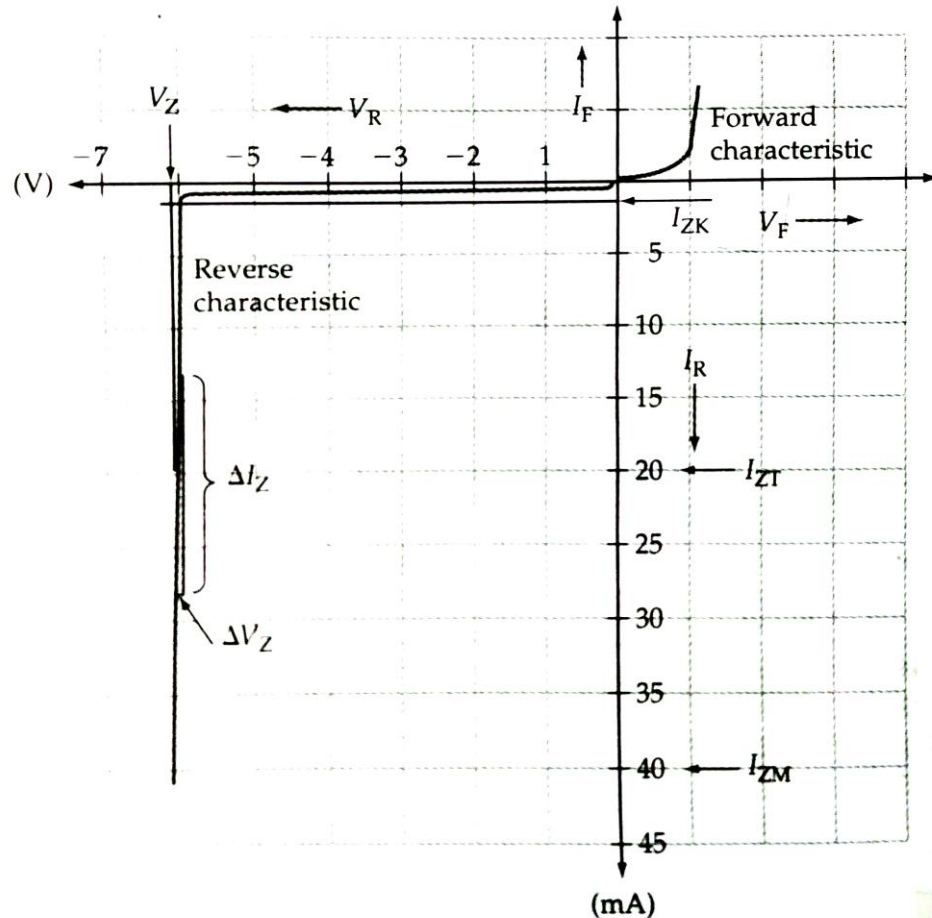
- Zener diode is a diode designed specially for operation in reverse breakdown.
 - Also called *breakdown diode*.
- Zener diodes have a breakdown voltage that remains extremely stable over a wide range of current levels.
- This property makes Zener diodes suitable for many useful applications as a voltage reference source.

Zener Diode



Circuit symbol of Zener diode

Zener Characteristics and Parameters



V_Z → Zener breakdown voltage

I_{ZT} → Test current for measuring V_Z

I_{ZK} → Reverse current near the knee of the characteristic, the minimum reverse current to sustain breakdown

I_{ZM} → Maximum Zener current, limited by the maximum power dissipation

Typical characteristics of a Zener diode

Zener Characteristics and Parameters

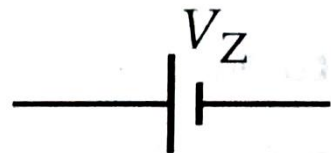
- Dynamic Impedance

$$Z_Z = \frac{\Delta V_Z}{\Delta I_Z}$$

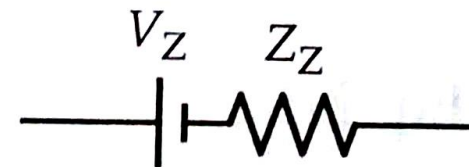
- Power Dissipation

$$P_D = V_Z I_{ZM}$$

- Equivalent Circuit



DC equivalent circuit



AC equivalent circuit

Zener Diode – Numerical Example

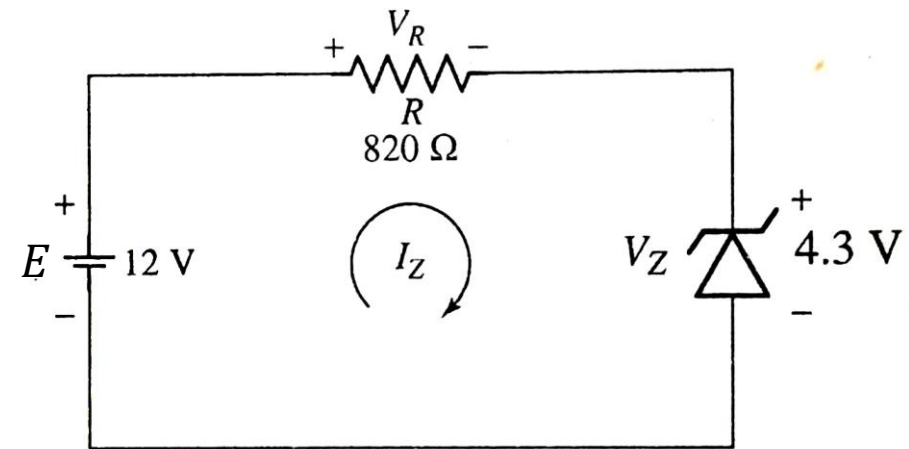
A 4.3 V Zener diode is connected in series with 820 Ω resistor and DC supply voltage of 12 V. Find the diode current and the power dissipation.

Solution:

Given,

$$V_Z = 4.3 \text{ V}, R = 820 \Omega, E = 12 \text{ V}$$

From the given parameters,
the circuit can be drawn as shown.



Zener Diode – Numerical Example

Using KVL for the circuit,

$$E = I_Z R + V_Z$$

Hence, diode current

$$\begin{aligned} I_Z &= \frac{E - V_Z}{R} \\ &= \frac{12\text{ V} - 4.3\text{ V}}{820\ \Omega} \end{aligned}$$

$$I_Z = 9.39\text{ mA}$$

Zener Diode – Numerical Example

Power dissipation

$$\begin{aligned}P_D &= V_Z I_Z \\ &= 4.3 \text{ V} \times 9.39 \text{ mA}\end{aligned}$$

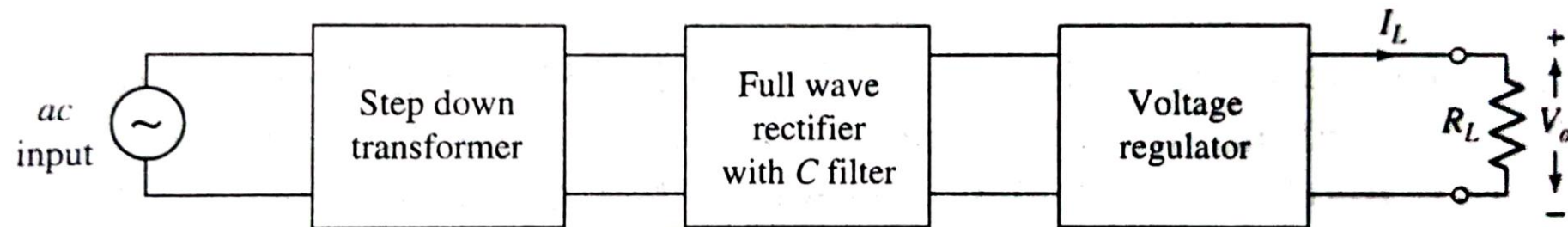
$$P_D = 40.377 \text{ mW}$$

Voltage Regulator

Voltage Regulator

- A voltage regulator is a circuit which accepts unregulated dc as input and provides a constant dc output voltage irrespective of changes in the line voltage and the load current.
- The output of a full wave rectifier with capacitor filter may be called unregulated dc since it varies with changes in load current and line voltage.
- Most of the electronic circuits require a stable dc voltage for their proper operation.
- Hence, it is necessary to regulate the output of full wave rectifier with filter.

Voltage Regulator



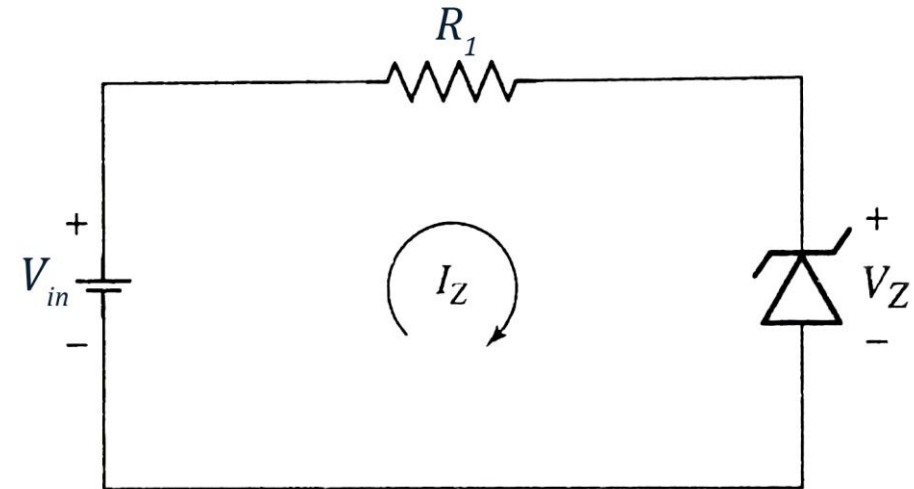
Block diagram of regulated dc power supply

Zener Diode as a Voltage Regulator

- Since Zener diodes have a breakdown voltage that remains extremely stable over a wide range of current levels, they can be used as voltage regulators.
- The regulators can be with a load or without a load.

Regulator with No Load

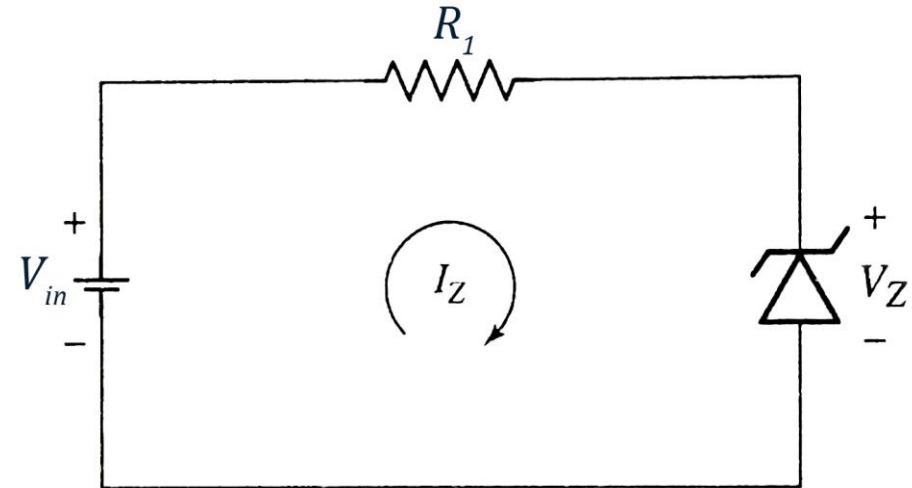
- Zener diode can be used as a voltage regulator as shown in the figure.
- This circuit does not have a load resistance and is normally used as a voltage reference source that supplies only a very low current (much lower than I_Z) to the output.



Regulator with No Load

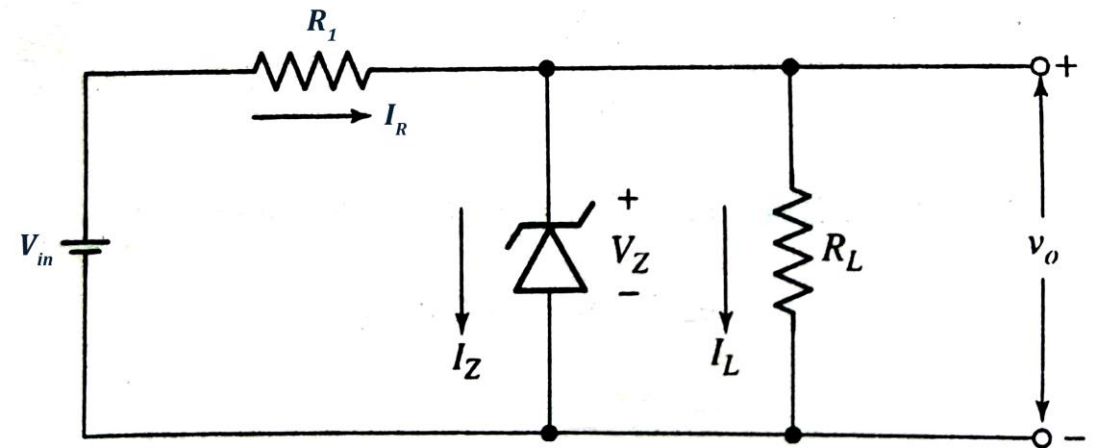
- Resistor R_1 limits the Zener diode current to the desired level.
- From the circuit,

$$I_Z = \frac{V_{in} - V_Z}{R_1}$$



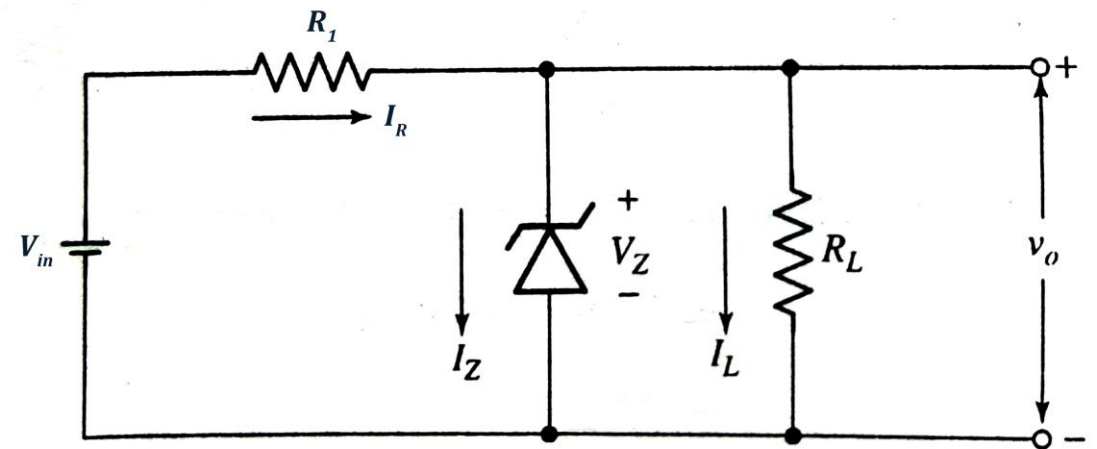
Regulator with a Load (Loaded Regulator)

- In this circuit, a load resistor is connected in parallel with the Zener diode.
- The total supply current (flowing through R_1) is the sum of Zener current I_Z and load current I_L .



Regulator with a Load (Loaded Regulator)

- Care must be taken to ensure that the minimum Zener current is large enough to keep the diode in reverse breakdown.
- Typically, $I_{Z(min)} = 5 \text{ mA}$ for a Zener diode with an I_{ZT} of 20 mA .



Regulator with a Load (Loaded Regulator)

- Since load resistor R_L and Zener diode are in parallel, the voltage across R_L is equal to the voltage across Zener diode.

i.e.,
$$V_o = V_Z \quad (1)$$

- So V_o remains constant even if V_{in} happens to change due to the fluctuations in ac line voltage.

Regulator with a Load (Loaded Regulator)

From the circuit,

$$I_R = I_Z + I_L \quad (2)$$

$$I_Z = I_R - I_L \quad (3)$$

Also,

$$I_R = \frac{V_{in} - V_o}{R_1} \quad (4)$$

Using Eqn. (4) in (3),

$$I_Z = \left[\frac{V_{in} - V_o}{R_1} \right] - I_L \quad (5)$$

Regulator with a Load (Loaded Regulator)

- Assume that V_{in} varies between $V_{in(min)}$ and $V_{in(max)}$ and I_L varies from $I_{L(min)}$ to $I_{L(max)}$.
- From Eqn. (5), we find that minimum Zener current flows when $V_{in} = V_{in(min)}$ and $I_L = I_{L(max)}$.
- The current through the Zener diode must be more than $I_{Z(min)}$, where $I_{Z(min)}$ is the minimum Zener current required to operate in the breakdown region.

$$\therefore \left[\frac{V_{in(min)} - V_o}{R_1} \right] - I_{L(max)} > I_{Z(min)} \quad (6)$$

Regulator with a Load (Loaded Regulator)

- Also, maximum Zener current flows when $V_{in} = V_{in(max)}$ and $I_L = I_{L(min)}$.
- The current through the Zener diode must be less than $I_{Z(max)}$, where $I_{Z(max)}$ is the maximum allowable Zener current for safe operation.

$$\therefore \left[\frac{V_{in(max)} - V_o}{R_1} \right] - I_{L(min)} < I_{Z(max)} \quad (7)$$

Voltage Regulator – Numerical Example 1

A Zener diode has a breakdown voltage of 10 V. It is supplied from a voltage source varying between 20–40 V in series with a resistance of 820 Ω . Using an ideal Zener model, obtain the minimum and maximum Zener currents.

Solution:

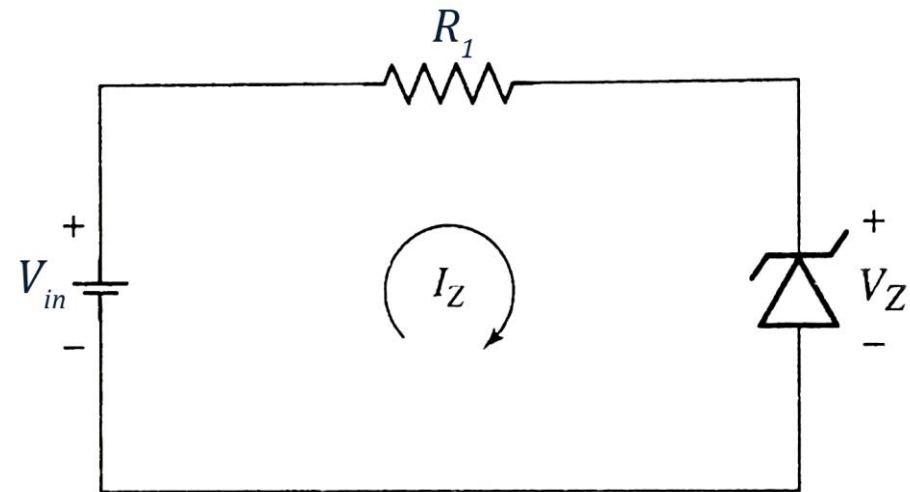
$$\text{Given } V_Z = 10 \text{ V}$$

$$V_{in} = 20 - 40 \text{ V}$$

That means,

$$V_{in(min)} = 20 \text{ V and } V_{in(max)} = 40 \text{ V}$$

$$R_1 = 820 \Omega$$



Voltage Regulator – Numerical Example 1

Considering ideal Zener model,

$$I_{Z(min)} = 0 \text{ mA}$$

For maximum value,

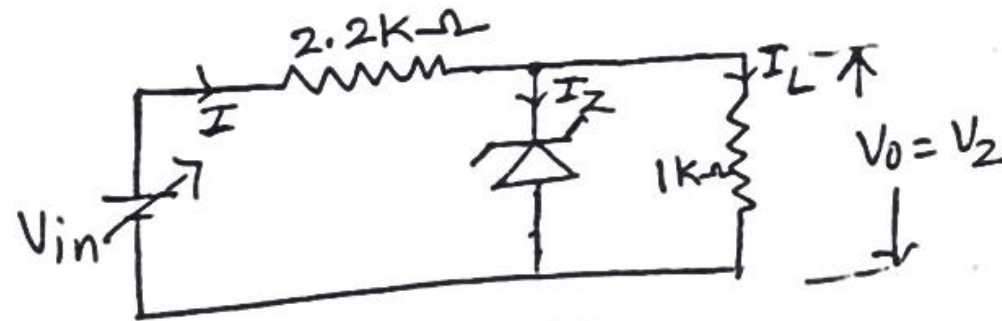
$$I_Z = \frac{V_{in} - V_Z}{R_1}$$

$$\begin{aligned} I_{Z(max)} &= \frac{V_{in(max)} - V_Z}{R_1} \\ &= \frac{40 \text{ V} - 10 \text{ V}}{820 \text{ V}} \end{aligned}$$

$$I_{Z(max)} = 36.5855 \text{ mA}$$

Voltage Regulator – Numerical Example 2

For a Zener regulator shown in the figure, calculate the range of input voltage for which output will remain constant.



$$V_Z = 6.1 \text{ V}, I_{Zmin} = 2.5 \text{ mA}, I_{Zmax} = 25 \text{ mA}, r_Z = 0 \Omega.$$

Solution:

$$\text{Given } R_1 = 2.2 \text{ k}\Omega, R_L = 1 \text{ k}\Omega, V_o = V_Z = 6.1 \text{ V}$$

Voltage Regulator – Numerical Example 2

The load current

$$\begin{aligned} I_L &= \frac{V_o}{R_L} \\ &= \frac{6.1 \text{ V}}{1 \text{ k}\Omega} \\ I_L &= 6.1 \text{ mA} \end{aligned}$$

From the circuit,

$$I = I_Z + I_L$$

Voltage Regulator – Numerical Example 2

$$\begin{aligned}I_{min} &= I_{Zmin} + I_L \\ &= 2.5 \text{ mA} + 6.1 \text{ mA}\end{aligned}$$

$$I_{min} = 8.6 \text{ mA}$$

And

$$\begin{aligned}I_{max} &= I_{Zmax} + I_L \\ &= 25 \text{ mA} + 6.1 \text{ mA}\end{aligned}$$

$$I_{max} = 31.1 \text{ mA}$$

Voltage Regulator – Numerical Example 2

Now,

$$V_{in} = IR_1 + V_Z$$

Minimum value

$$\begin{aligned} V_{in(min)} &= I_{min}R_1 + V_Z \\ &= 8.6 \text{ mA} \times 2.2 \text{ k}\Omega + 6.1 \text{ V} \\ &= 18.92 \text{ V} + 6.1 \text{ V} \\ V_{in(min)} &= 25.02 \text{ V} \end{aligned}$$

Voltage Regulator – Numerical Example 2

Minimum value

$$\begin{aligned}V_{in(max)} &= I_{max}R_1 + V_Z \\ &= 31.1 \text{ mA} \times 2.2 \text{ k}\Omega + 6.1 \text{ V} \\ &= 68.42 \text{ V} + 6.1 \text{ V}\end{aligned}$$

$$V_{in(max)} = 74.52 \text{ V}$$

Hence the output will remain constant for the range of input voltage

$$25.02 \text{ V} < V_{in} < 74.52 \text{ V}$$

Voltage Regulator – Numerical Example 3

For the circuit shown in the figure, find current and voltages in the circuit for $R_L = 450 \Omega$, $V_Z = 10 \text{ V}$.



Solution:

Given $V_{in} = 20 \text{ V}$, $V_Z = 10 \text{ V}$

$R_1 = 200 \Omega$, $R_L = 450 \Omega$

Voltage Regulator – Numerical Example 3

The output voltage

$$V_o = V_Z$$

$$V_o = 10 V$$

The voltage across R_1

$$V_R = V_{in} - V_Z$$

$$= 20 V - 10 V$$

$$V_R = 10 V$$

Voltage Regulator – Numerical Example 3

Current through R_1

$$\begin{aligned} I_R &= \frac{V_R}{R_1} \\ &= \frac{10\text{ V}}{200\ \Omega} \\ I_R &= 50\text{ mA} \end{aligned}$$

Voltage Regulator – Numerical Example 3

Load current

$$I_L = \frac{V_o}{R_L}$$
$$= \frac{10 \text{ V}}{450 \Omega}$$

$$I_L = 22.22 \text{ mA}$$

Zener current

$$I_Z = I_R - I_L$$
$$= 50 \text{ mA} - 22.22 \text{ mA}$$

$$I_Z = 27.78 \text{ mA}$$

Voltage Regulator – Numerical Example 4

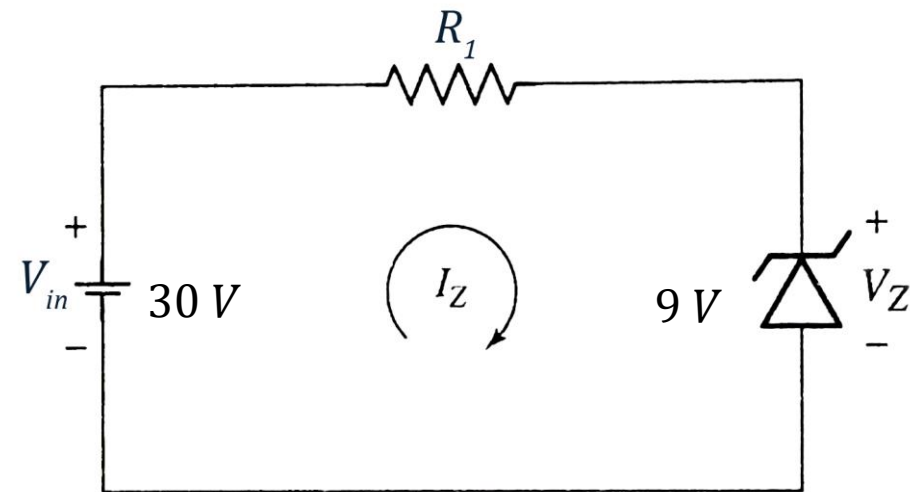
A 9 V reference source is to use a series connected Zener diode and a resistor connected to 30 V supply. If Zener diode with $V_Z = 9\text{ V}$, $I_{ZT} = 20\text{ mA}$ is selected, then determine the value of series resistance and calculate the circuit current when the supply voltage drops to 27 V.

Solution:

Given $V_{in} = 30\text{ V}$, $V_o = 9\text{ V}$

$V_Z = 9\text{ V}$, $I_{ZT} = 20\text{ mA}$

From the given parameters,
the circuit can be drawn as shown.



Voltage Regulator – Numerical Example 4

From the circuit,

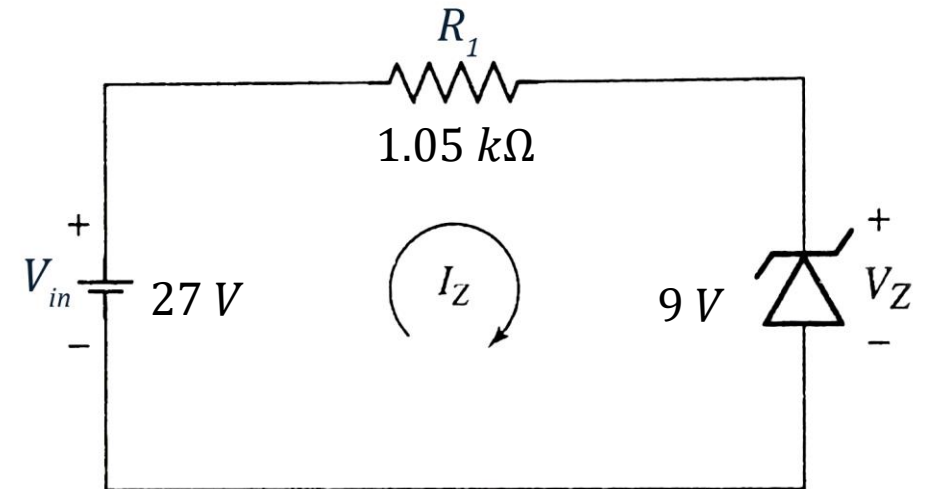
$$\begin{aligned} R_1 &= \frac{V_{in} - V_Z}{I_Z} \\ &= \frac{30\text{ V} - 9\text{ V}}{20\text{ mA}} \\ R_1 &= 1.05\text{ k}\Omega \end{aligned}$$

Voltage Regulator – Numerical Example 4

Now if $V_{in} = 27\text{ V}$,

$$I_Z = \frac{V_{in} - V_Z}{R_1}$$
$$= \frac{27\text{ V} - 9\text{ V}}{1.05\text{ k}\Omega}$$

$$I_Z = 17.14\text{ mA}$$



Voltage Regulator – Numerical Example 5

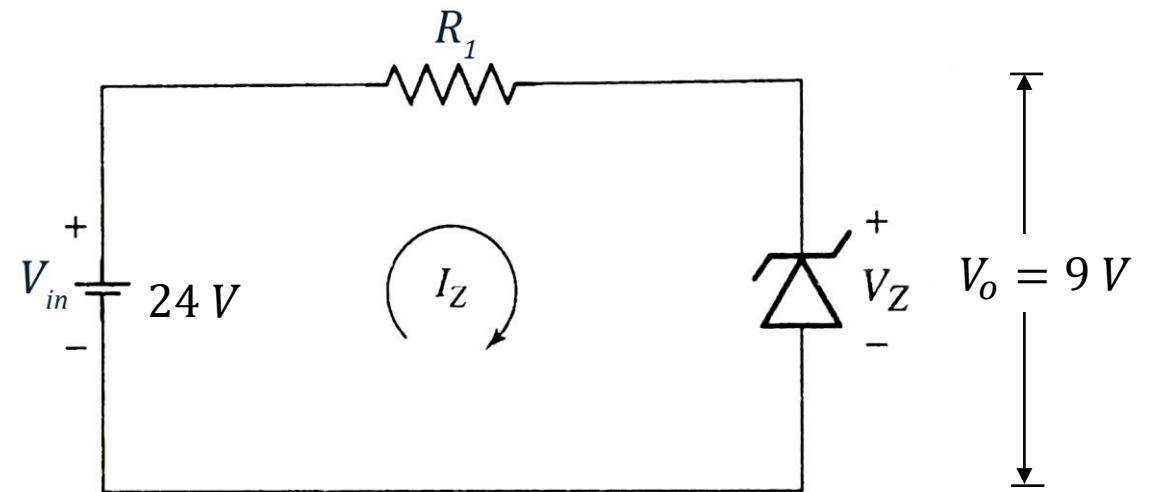
Design a 9 V DC reference source consisting of a Zener diode and series connected resistor to operate from a 24 V supply [$I_{ZT} = I_Z = 20 \text{ mA}$].

Solution:

Given $V_{in} = 24 \text{ V}$, $V_o = 9 \text{ V}$

$I_{ZT} = 20 \text{ mA}$

From the given parameters,
the required circuit is as shown.



Voltage Regulator – Numerical Example 5

Select a suitable Zener diode whose breakdown voltage

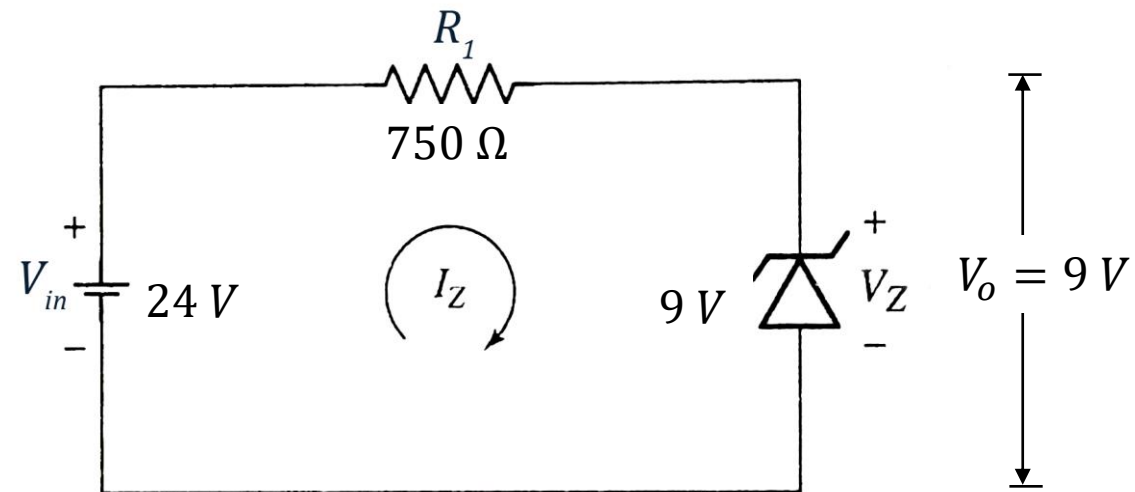
$$V_Z = V_o = 9V$$

To find R_1

$$\begin{aligned} R_1 &= \frac{V_{in} - V_Z}{I_Z} \\ &= \frac{24V - 9V}{20mA} \\ R_1 &= 750 \Omega \end{aligned}$$

Voltage Regulator – Numerical Example 5

The designed circuit can be drawn as below:



Voltage Regulator – Numerical Example 6

Design Zener voltage regulator for the following specifications:

Input Voltage = $10\text{ V} \pm 20\%$, Output Voltage = 5 V , $I_L = 20\text{ mA}$, $I_{Z(\min)} = 5\text{ mA}$ and $I_{Z(\max)} = 80\text{ mA}$.

Solution:

Given $V_{in} = 10\text{ V} \pm 20\%$

$$20\% \text{ of } 10\text{ V} = \frac{20}{100} \times 10\text{ V} = 2\text{ V}$$

$$\text{Hence } V_{in(\min)} = 10\text{ V} - 2\text{ V} = 8\text{ V}$$

$$\text{and } V_{in(\max)} = 10\text{ V} + 2\text{ V} = 12\text{ V}$$

Also given $V_o = 5\text{ V}$

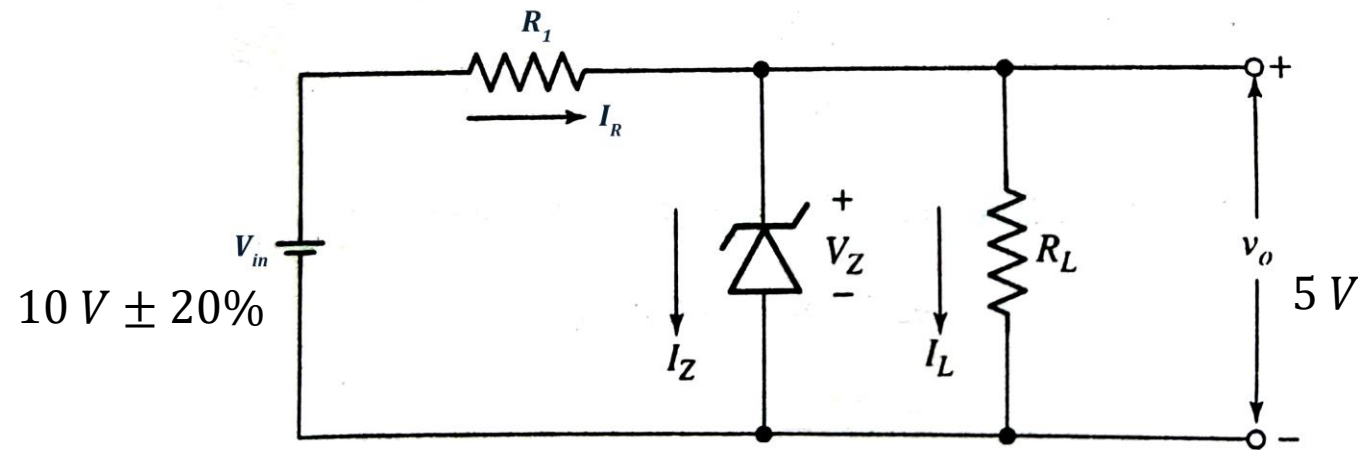
$$I_L = 20\text{ mA.}$$

$$I_{Z(\min)} = 5\text{ mA}$$

$$I_{Z(\max)} = 80\text{ mA}$$

Voltage Regulator – Numerical Example 6

From the given parameters, the required circuit can be drawn as below:



Voltage Regulator – Numerical Example 6

Select a suitable Zener diode whose breakdown voltage

$$V_Z = V_o = 5V$$

To find R_L

$$\begin{aligned} R_L &= \frac{V_o}{I_L} \\ &= \frac{5V}{20mA} \\ R_L &= 250\ \Omega \end{aligned}$$

Voltage Regulator – Numerical Example 6

To find R_1 :

From the circuit, $I_R = I_Z + I_L$

For maximum value of R_1 :

I_Z will be $I_{Z(min)}$ if $V_{in} = V_{in(min)}$

Then

$$\begin{aligned} I_R &= I_{Z(min)} + I_L \\ &= 5 \text{ mA} + 20 \text{ mA} \end{aligned}$$

$$I_R = 25 \text{ mA}$$

Voltage Regulator – Numerical Example 6

Then

$$V_{in(min)} = I_R R_{1(max)} + V_Z$$

$$R_{1(max)} = \frac{V_{in(min)} - V_Z}{I_R}$$

$$R_{1(max)} = \frac{8\text{ V} - 5\text{ V}}{25\text{ mA}}$$

$$R_{1(max)} = 120\ \Omega$$

Voltage Regulator – Numerical Example 6

For minimum value of R_1 :

I_Z will be $I_{Z(max)}$ if $V_{in} = V_{in(max)}$

Then

$$\begin{aligned} I_R &= I_{Z(max)} + I_L \\ &= 80 \text{ mA} + 20 \text{ mA} \end{aligned}$$

$$I_R = 100 \text{ mA}$$

Voltage Regulator – Numerical Example 6

Then

$$V_{in(max)} = I_R R_{1(min)} + V_Z$$

$$R_{1(min)} = \frac{V_{in(max)} - V_Z}{I_R}$$

$$R_{1(min)} = \frac{12\text{ V} - 5\text{ V}}{100\text{ mA}}$$

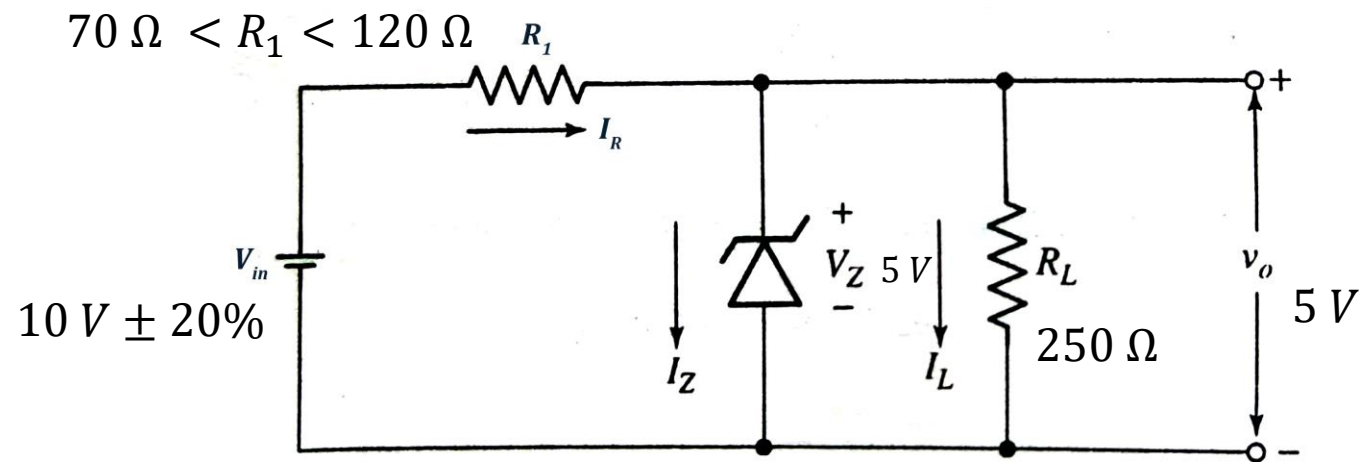
$$R_{1(min)} = 70\ \Omega$$

Hence

$$70\ \Omega < R_1 < 120\ \Omega$$

Voltage Regulator – Numerical Example 6

The designed circuit can be drawn as below:



Voltage Regulator – Numerical Example 7

Design a Zener diode voltage regulator circuit to meet the following specifications:

$$I_L = 20 \text{ mA}, V_O = 5 \text{ V}, P_Z = 500 \text{ mW}, V_i = 12 \text{ V} \pm 2 \text{ V} \text{ and } I_{Z\min} = 8 \text{ mA}.$$

Solution:

$$\text{Given } V_{in} = 12 \text{ V} \pm 2 \text{ V}$$

$$\text{Hence } V_{in(\min)} = 12 \text{ V} - 2 \text{ V} = 10 \text{ V}$$

$$\text{and } V_{in(\max)} = 12 \text{ V} + 2 \text{ V} = 14 \text{ V}$$

$$\text{Also given } V_O = 5 \text{ V}$$

$$I_L = 20 \text{ mA}.$$

$$I_{Z(\min)} = 8 \text{ mA}$$

$$P_Z = 500 \text{ mW}$$

Voltage Regulator – Numerical Example 7

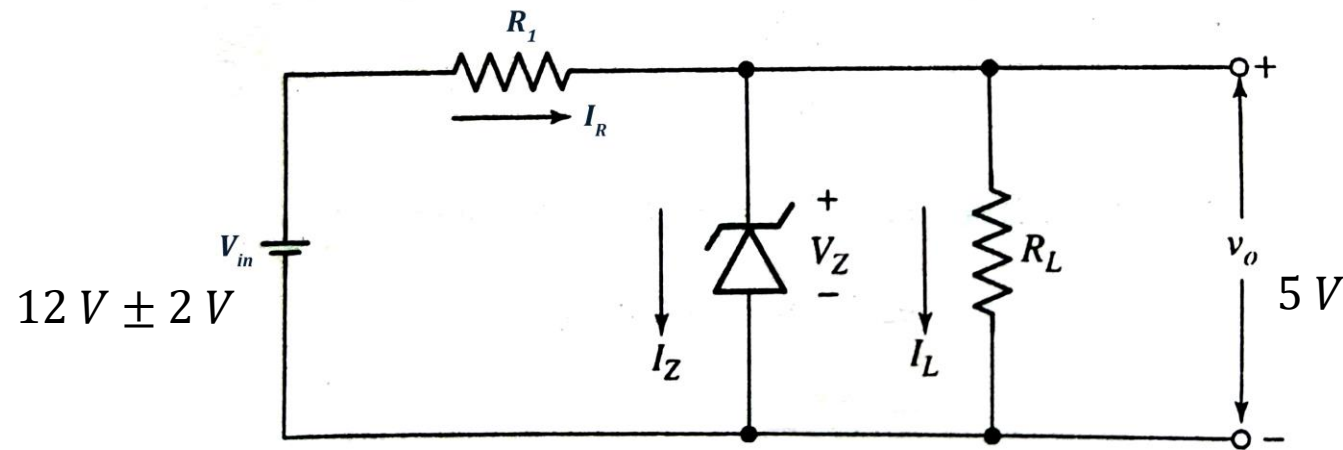
$$P_Z = V_Z I_{Z(max)}$$

$$I_{Z(max)} = \frac{P_Z}{V_Z}$$
$$= \frac{500 \text{ mW}}{5 \text{ V}}$$

$$I_{Z(max)} = 100 \text{ mA}$$

Voltage Regulator – Numerical Example 7

From the given parameters, the required circuit can be drawn as below:



Voltage Regulator – Numerical Example 7

Select a suitable Zener diode whose breakdown voltage

$$V_Z = V_o = 5V$$

To find R_L

$$\begin{aligned} R_L &= \frac{V_o}{I_L} \\ &= \frac{5V}{20mA} \\ R_L &= 250\ \Omega \end{aligned}$$

Voltage Regulator – Numerical Example 7

To find R_1 :

From the circuit, $I_R = I_Z + I_L$

For maximum value of R_1 :

I_Z will be $I_{Z(min)}$ if $V_{in} = V_{in(min)}$

Then

$$\begin{aligned} I_R &= I_{Z(min)} + I_L \\ &= 8 \text{ mA} + 20 \text{ mA} \end{aligned}$$

$$I_R = 28 \text{ mA}$$

Voltage Regulator – Numerical Example 7

Then

$$V_{in(min)} = I_R R_{1(max)} + V_Z$$

$$R_{1(max)} = \frac{V_{in(min)} - V_Z}{I_R}$$

$$R_{1(max)} = \frac{10\text{ V} - 5\text{ V}}{28\text{ mA}}$$

$$R_{1(max)} = 178.57\ \Omega$$

Voltage Regulator – Numerical Example 7

For minimum value of R_1 :

I_Z will be $I_{Z(max)}$ if $V_{in} = V_{in(max)}$

Then

$$\begin{aligned} I_R &= I_{Z(max)} + I_L \\ &= 100 \text{ mA} + 20 \text{ mA} \end{aligned}$$

$$I_R = 120 \text{ mA}$$

Voltage Regulator – Numerical Example 7

Then

$$V_{in(max)} = I_R R_{1(min)} + V_Z$$

$$R_{1(min)} = \frac{V_{in(max)} - V_Z}{I_R}$$

$$R_{1(min)} = \frac{14\text{ V} - 5\text{ V}}{120\text{ mA}}$$

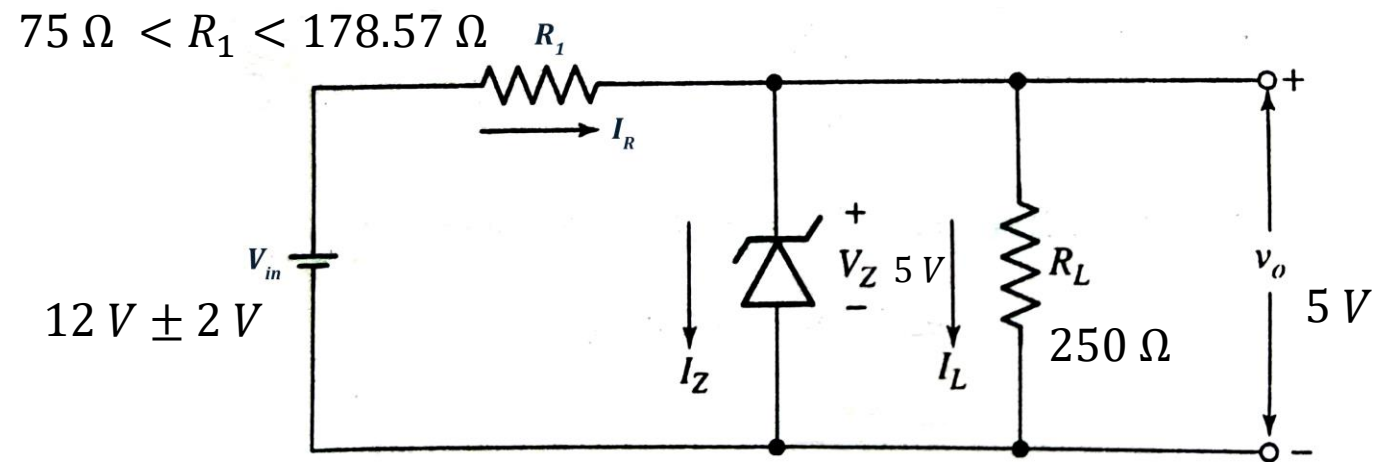
$$R_{1(min)} = 75\ \Omega$$

Hence

$$75\ \Omega < R_1 < 178.57\ \Omega$$

Voltage Regulator – Numerical Example 7

The designed circuit can be drawn as below:



Regulator Performance

Source Effect and Line Regulation

- *Source effect* is defined as the change in the output voltage for 10% change in the source voltage.

$$\text{Source effect} = \Delta V_o \text{ for a 10\% change in } V_s$$

- If this change is expressed as a percentage of the output voltage, it is called *line regulation*.

$$\text{Line regulation} = \frac{(\Delta V_o \text{ for a 10\% change in } V_s) \times 100\%}{V_o}$$

Load Effect and Load Regulation

- *Load effect* is defined as the change in the output voltage when the load current is increased from zero to its maximum level

$$\text{Load effect} = \Delta V_o \text{ for } \Delta I_{L(max)}$$

- If this change is expressed as a percentage of the output voltage, it is called *load regulation*.

$$\text{Load regulation} = \frac{(\Delta V_o \text{ for } \Delta I_{L(max)}) \times 100\%}{V_o}$$

Regulator Performance

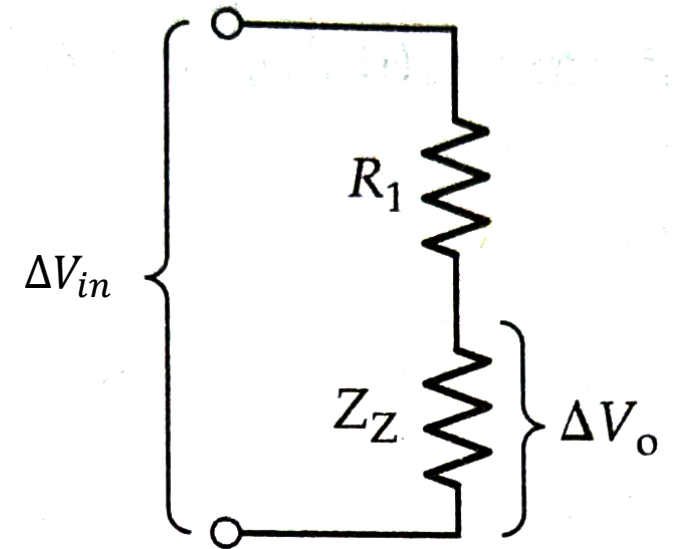
- The performance of a regulator can be expressed in terms of the source and load effects and line and load regulations.
- If there is an input ripple voltage, it will be severely attenuated.
- The *ripple rejection ratio* is the ratio of the output to input ripple amplitudes.

$$RRR = \frac{V_{ro}}{V_{ri}}$$

Regulator Performance

- The ac equivalent circuit is drawn by replacing the diode with its dynamic impedance Z_Z .
- Consider an equivalent circuit of a regulator with no load.
- When the input voltage changes by ΔV_{in} , the change in the output voltage is

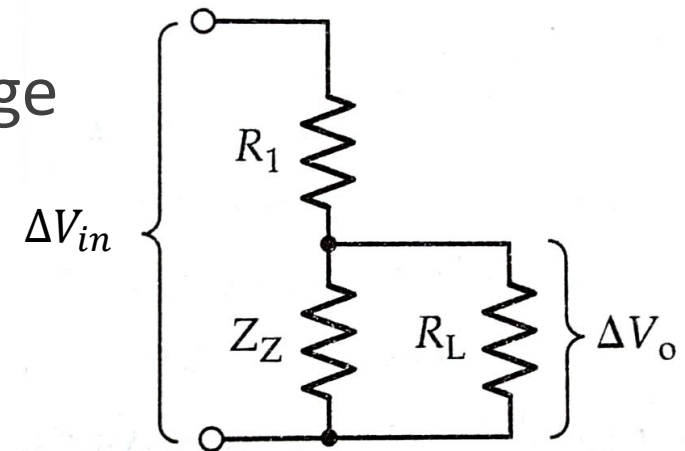
$$\Delta V_o = \frac{\Delta V_{in} \times Z_Z}{R_1 + Z_Z}$$



Regulator Performance

- Consider an equivalent circuit of a regulator with a load.
- When the input voltage changes by ΔV_{in} , the change in the output voltage is

$$\Delta V_o = \frac{\Delta V_{in} \times (Z_Z || R_L)}{R_1 + (Z_Z || R_L)}$$



Regulator Performance

- The ripple rejection ratio can be calculated by substituting input ripple amplitude V_{ri} for input voltage change and output ripple amplitude V_{ro} for output voltage change.

- For a regulator without load,

$$RRR = \frac{V_{ro}}{V_{ri}} = \frac{Z_Z}{R_1 + Z_Z}$$

- For a loaded regulator,

$$RRR = \frac{V_{ro}}{V_{ri}} = \frac{(Z_Z || R_L)}{R_1 + (Z_Z || R_L)}$$

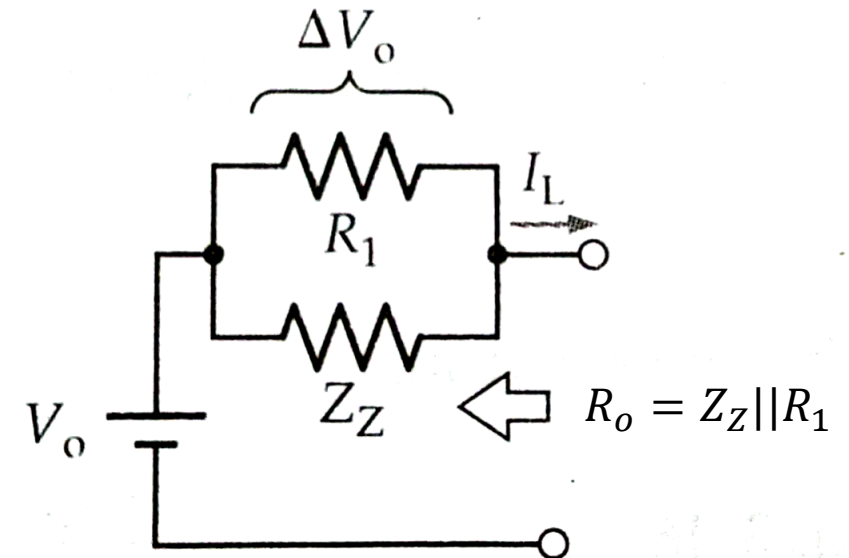
Regulator Performance

- To calculate the load effect of the regulator, the circuit output resistance has to be calculated.
- From the Thevenin equivalent circuit, assuming a zero source resistance, the circuit output resistance is

$$R_o = Z_Z || R_1$$

- When the load current changes by ΔI_L , the output voltage change is

$$\Delta V_o = \Delta I_L \times (Z_Z || R_1)$$



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